Department of Education and Early Childhood Development

Mental computation and estimation

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Traditionally, much of the focus of school mathematics has been on teaching algorithms for arithmetic calculation. However, mental computation and estimation are just as important for everyday life as well as to enhance the learning of mathematics. In everyday life it is very common that an approximate answer to an arithmetic problem is needed, rather than an exact one. This is especially so if the answer can be found quickly, without resorting to tools such as pencil and paper or a calculator or the shop's cash register. When an exact answer is required, it is often obtained by using a calculator or cash register, and students should be taught the importance of mentally estimating answers as a check for reasonableness of these answers. In addition, research over the last several decades has shown that students who are encouraged to use efficient mental computation strategies develop deeper understanding of number relationships. It is important, then, that children learn to apply efficient mental computation and estimation strategies.

The term *estimation* is used here to refer to mental estimation of the result of a calculation, rather than the equally important ability to estimate measured quantities, such as length or volume.

Facts about mental computation

- Many people work out strategies for mental computation from their good understanding of place value, their number sense and their understanding of the meaning of the arithmetic operation and its properties. Many children can compute mentally *before* they learn the relevant formal written algorithms at school.
- People good at mental computation use a wide variety of methods, for example, making use of the distributive law, factors, halving and doubling.
- The methods used for mental computation are often quite different from the paper-and-pencil algorithms taught at school. Hope and Sherrill (1987) found that efficient strategies used by expert mental calculators
    - generally eliminate the need for 'carrying',
    - often proceed in a left-to-right manner
    - often progressively incorporate interim calculations into a single result For example to calculate $9\times$ 742, they may start with the hundreds "9 times 700 is 6300", incorporate interim calculations progressively by adding "9 forties" to get 6660 and then adding "9 twos", to get 6678. This eliminated the need for 'carrying' within the multiplication





- Amongst students who were accurate with mental computation Heirdsfield and Cooper (2004) found that some students were able to choose from a variety of strategies based on their number sense, whereas others tended to use mental images of paper-and-pencil algorithms. Whilst the algorithmic approach to mental computation may serve some students well with simple calculations, this approach does not allow them to move on to more complex mental computation (see for example, Callingham, 2005).
- In general, less competent students use less efficient strategies (such as counting on by ones rather than by tens) and they use them for longer. Focussed teaching is needed to help them move on.
- People good at mental computation select strategies which do not make high demands on short-term memory. This is why short term memory capacity does not correlate highly with proficiency in mental computation (see Hope & Sherrill, 1987).
- A good knowledge of number facts is essential for efficient mental computation as this reduces the demands on short-term memory. Hope and Sherrill (1987) found that the expert mental calculators had near perfect recall of basic multiplication facts, and could also recall larger numerical equivalents such as $13^2, 15^2, 25^2$
- Teaching rules such as “add a zero to multiply by ten” without understanding is dangerous because they are misused by all but the best students.
- Some mental strategies are cognitively easier than others to understand and to create. For example, breaking one number into constituent parts as in decomposition subtraction is cognitively easier than changing two numbers to an equivalent calculation. (This is one reason why the principles behind decomposition subtraction are easier to understand than the principles behind equal additions subtraction).

Characteristics of mental methods

- Mental methods are often varied to take advantage of known properties of the actual numbers in the problem. For example, mental methods use facts such as 8 is close to 10, 25 is one quarter of 100 or 6 and 4 add to 10. Favourite number combinations are often used as a basis of computation.
- Many mental methods follow unconventional patterns like subtracting or multiplying from left to right so that the big quantities are dealt with first (e.g. hundreds before ones). This is advantageous when an estimate, rather than a precise answer, is enough. In real life, estimation is as important a skill as exact calculation. It is a skill essential to complement calculator use.
- It is common in mental computation to modify the question and then compensate later (e.g. by rounding, doubling, halving, etc).
- Mental methods are often based on using round numbers (e.g. 600, 1400, 30). In contrast, some formal written algorithms are hard to carry out with round numbers (think about 1000 – 657 done by a formal subtraction algorithm). Children make many mistakes dealing with zero in formal written algorithms.
- Mental computation is often step-by-step, rather than dealing with all the relationships in the problem simultaneously.
- Mental computation sometimes uses a ‘primitive’ version of an operation. For example, addition may be done by counting on, multiplication may be done by repeated addition, e.g. 3×150 is 150 + 150 + 150 = 300 + 150 = 450.

- For many people, the types of numbers that can be dealt with by mental computation are limited. For example, many people can calculate with 1/2 but not with other fractions.

Teaching mental computation – general principles

The excellent resources by Alistair McIntosh (2005) and McIntosh, de Nardi and Swan (1994) provide advice, as well as games and activities:

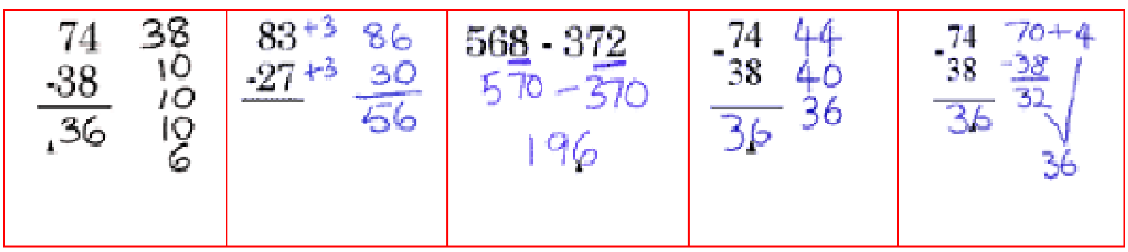
- Teach mental computation, don't just test it. Emphasise how answers are obtained: don't put all the emphasis just on speed or the correct answers. Tests of mental arithmetic that emphasise speed alone tend to increase mathematics anxiety.
- Teachers should learn about the strategies that children use and learn how to describe mental strategies to children.
- In the middle grades, offer 10-15 minute sessions a few times a week, involving class sharing, instruction and practice with well chosen games and activities that build fluency.
- Class discussion is important for sharing mental methods among students. Even the weaker students have interesting methods.
- Some strategies can be taught through class discussion, explanation and practice. Be wary of including rules to learn by rote (e.g. adding zeros) since they are almost invariably misused by all but the most competent.
- Value creativity, exploration, efficiency and inventiveness.
- Encourage mental methods before, as well as after, written computation.

Heirdsfield, Cooper and Irons (1999) summarise the key features of a good program for teaching mental computation as:

- variety
- individuality
- emphasis on number sense
- building understanding of place value and other arithmetic principles.

Sowder (1990, p.19) asserts that "mental computation should not be delayed until after formal written algorithms have been mastered. In fact delaying it until that time encourages students to mentally use the algorithms meant only for pencil-and-paper calculations."

Many examples of mental methods that students commonly use, and can learn to use, are given by Stacey, Varughese and Marston (2003). For subtraction, this resource explains five general methods and gives small movies of students' talking through what they are doing. The images below show part of the students' working, when they are using complementary addition, subtraction in stages, rounding, equal additions principle, and the renaming principle, all of which are explained in the resource.



Mental computation and written algorithms

Many children will be able to do mental computations before you teach them written computation. In a classroom emphasising appropriate number activities, children will invent and refine their own methods of calculation. We should be cautious of the fact that teaching of algorithms may result in children giving up their own numerical thinking and becoming dependent on others. Kamii and Dominick (1997) note that “when we try to teach children to make relationships between numbers (logico-mathematical knowledge) by teaching them algorithms (social-conventional knowledge), we redirect their attention from trying to make sense of numbers to remembering procedures” (p. 59).

Teaching written algorithms needs to support children to develop their understanding of number relationships. Gravemeijer (2003) is critical of teaching algorithms “in ready-made form” that students do not understand, advocating instead “instructional sequences in which the students act like mathematicians of the past and reinvent procedures and algorithms” (p. 121) as a means of promoting growth in mathematical understanding.

Kamii reports on the good results of classroom experiments where children invent their own methods of arithmetic, and also gives many examples of the classroom activities they used. The overarching principle is to have children construct their understanding of mathematics themselves, rather than internalise what others present to them (see for example, Kamii, 2000; Kamii & Dominick, 1997).

Some researchers even argue that students should not be taught algorithms, but should invent their own methods instead. Of course, this would need to be carefully managed by teachers, so that students do not practise wrong methods or inadequate methods that only work in special cases. This argument against teaching algorithms has been put most strongly by Constance Kamii (see for example, Kamii & Dominick, 1997). Squarely in the mainstream, however, is the substantial body of thought, developed over many years, that schools should put most emphasis on having students use mental calculation and estimation for easy to moderate calculations and teach sensible and careful calculator use for harder calculations. See, for example, Shuard (1991).

Mental estimation and calculator use

Students should be taught to first estimate an answer to a problem before using their calculator then to check their procedure if there is a large discrepancy. Sowder (1990) asserts that an understanding of numbers is “fundamental to expert calculator use” and that “an increased emphasis on mental computation of whole numbers will do much to develop the number sense needed to understand arithmetic, to estimate, to deal with technology” (p. 20). Levin (1981) also stresses the importance of developing students’ estimation strategies to verify the reasonableness of calculator-generated answers.

References

Callingham, R. (2005). Primary students' mental computation: Strategies and achievement. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice (Proceedings of 28th annual conference of the Mathematics Education Research Group of Australasia*, pp. 193–200). Melbourne: MERGA.

Gravemeijer, K. (2003). Facts and algorithms as products of students' own mathematical activity. In Kilpatrick, J., Martin, W. G., & Schifter, D. (Eds.), *A research companion to Principles and Standards for school mathematics*, pp. 114–122. Reston, VA: National Council of Teachers of Mathematics.

Heirdsfield, A. M. & Cooper, T. J. (2004). Factors affecting the process of proficient mental addition and subtraction: case studies of flexible and inflexible computers. *Journal of Mathematical Behavior*, 23, 443–463.

Heirdsfield, A. M., Cooper, T. J. & Irons, C. J. (1999) *Traditional pen-and-paper vs mental approaches to computation: the lesson of Adrien*. Australian Association for Research in Education (AARE) Conference, Melbourne Melbourne.. http://www.aare.edu.au/99pap/hei99610.htm

Hope, J. A. & Sherrill, J. M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education*, *18*(2), 98–111.

Levin, J. A. (1981). Estimation techniques for arithmetic: Everyday math and mathematics instruction. *Educational Studies in Mathematics*, *12*(4), 421–434.

Kamii, C. & Dominick, A. (1997). To teach or not to teach algorithms. *Journal of Mathematical Behavior*, *16*(1), 51–61.

Kamii, C. (2000). *Young children reinvent arithmetic: implications of Piaget's theory*, Teacher College Press: New York.

McIntosh, A. (2005). *Mental computation: a strategies approach*. Hobart: Department of Education, Tasmania.

McIntosh, A., De Nardi, E., & Swan, P. (1994). *Think mathematically! How to teach mental maths in the primary classroom*. Longman Cheshire: Melbourne.

Shuard, H. (1992). CAN: Calculator use in the primary grades in England and Wales. In J. Fey &C. Hirsch (Eds.), *Calculators in mathematics education* (pp. 33-45). Reston, Virginia: National Council of Teachers of Mathematics.

Sowder, J. T. (1990). Mental computation and number sense. *The Arithmetic Teacher*, *37*(7), 18–20.

Stacey, K., Varughese, N., Marston, K. (2003) *Teaching Mental and Written Computation.* (Version 1.1) University of Melbourne (CD-ROM). http://extranet.edfac.unimelb.edu.au/DSME/tmwc/