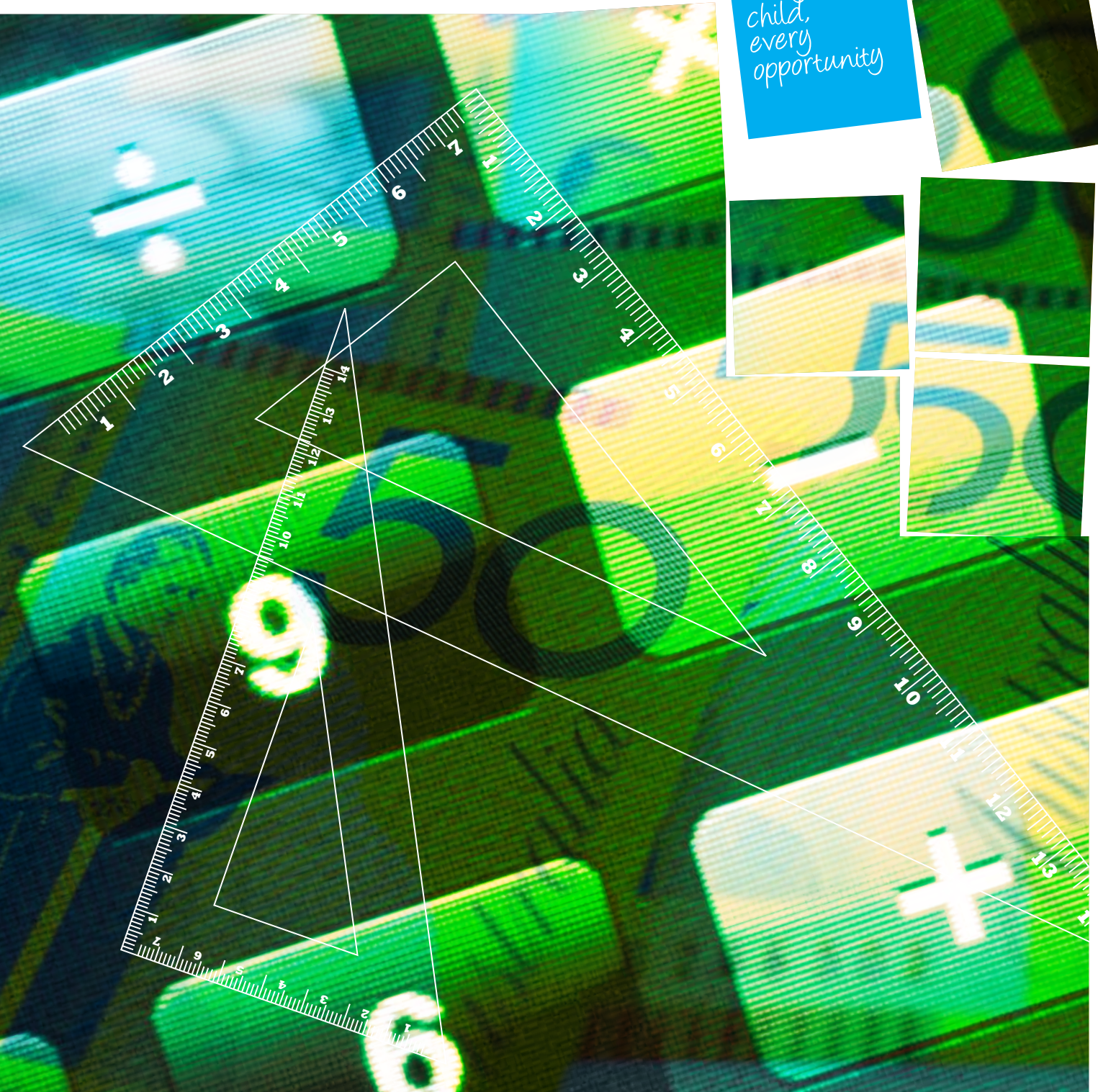


Numeracy in practice: teaching, learning and using mathematics

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Every
child,
every
opportunity



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Summary

Numeracy in Practice: Teaching, Learning and Using Mathematics is a research and policy-based resource for teachers of numeracy in Years P–10. It is an initiative of the Victorian Department of Education and Early Childhood Development, and intended to be a companion volume to *Literacy Teaching and Learning in Victorian Schools*.

The report is based on a focused literature search undertaken to identify and present findings from local and international research relevant to teaching, learning and using mathematics in the 21st century. It highlights those aspects that make a difference to student numeracy outcomes. *Numeracy in Practice: Teaching, Learning and Using Mathematics* focuses on the characteristics of effective numeracy teaching and addresses key classroom issues including: what to teach, how to teach numeracy, how to cater for diversity and how to make best use of technology. The report also focuses on the ways in which the school and the community can support effective teaching of mathematics.

Following an introduction, which examines why numeracy is a priority for all Victorian schools, eight sections explore key areas of research relevant to the teaching and learning of mathematics in Victorian schools.

Section 2 describes how numeracy is defined both locally and overseas and examines its relationship to mathematics. Numeracy is best described as a key outcome of how mathematics is taught and learned – it bridges the gap between mathematics learned at school and the variety of contexts where it needs to be used in everyday life.

Australia's performance in international assessment programs is discussed in section 3. While Australia consistently performs above the international average in Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) tests, the performance of some student groups is significantly lower.

Section 4 outlines approaches to numeracy in Australia informed by numeracy research projects undertaken over the past two decades. Section 5 discusses the features of effective numeracy teaching including the use of technology to support teaching practice and student learning, and the practices of schools achieving particular success in numeracy teaching.

Improving outcomes for under performing students is discussed in section 6 with examples of programs and teaching practices.

Section 7 examines the purposes of assessments and the varied approaches teachers can use to provide direction for teaching and curriculum planning.

Section 8 lists research-based resources provided by the Department to support teachers to implement the Mathematics domain of the Victorian Essential Learning Standards (the Standards). A key feature of these resources is that they place the learner at the centre of curriculum planning and delivery.

1. Introduction

The *Blueprint for Education and Early Childhood Development* (Department of Education and Early Childhood Development 2008a) recognises the need to increase efforts to improve Victoria's performance in numeracy, and achieving high levels of numeracy for all students is a key priority for all Victorian schools. One of the six goals of the Blueprint is to ensure that:

In Year 5, Victorian students will have the basic numeracy skills to succeed in mathematics, and support will be in place for those who do not.

Numeracy and literacy remain key domains of learning which are essential for success at school; provide a bridge to further study and work; and ensure children are well prepared for future economic and social prosperity.

Numeracy, a priority for all: Challenges for Australian schools (Department of Education, Science and Training 2000) expresses the importance of numeracy in terms of societal and individual needs:

Changing economic realities in the last 15 years, with increasing globalisation and use of technology, have seen the demand for unskilled labour fall markedly. This has prompted the recognition that high-quality basic education reflected in the knowledge, skills and attitudes of the workforce is vital for countries like Australia in maintaining national prosperity and social stability. The acquisition by all students of appropriate numeracy skills is now much more crucial than in the past. Numeracy, like literacy, provides key enabling skills for individuals to participate successfully in schooling. Furthermore, numeracy equips students for life beyond school, in providing access to further study or training, to personal pursuits and to participation in the world of work and in the wider community.

These arguments are endorsed by the National Numeracy Review commissioned by the Human Capital Working Group of COAG (2008). The final report of the review remarks that 'policy statements alone are not enough to secure improved numeracy outcomes', and that:

an important requirement for quality outcomes for education systems is that there is an appropriate alignment between national policy agreements, curriculum and assessment practices and classroom pedagogy (p. vii).

Promoting numeracy in all Victorian schools is a task for all who are involved with schools. It is a key task for those engaged in school leadership positions – principals and curriculum leaders – in developing school policies, in allocating resources and supporting teachers. Achieving better outcomes in numeracy is, of course, a key responsibility for teachers. School Councils too have an important role in supporting the development of appropriate policies and ensuring that achieving high standards of numeracy is incorporated into the school's cycle of goal setting, planning for school improvement and cyclical review of performance in these areas.

School leadership is vital for ensuring that appropriate policies and resources, together with effective coordination and teacher professional development, are provided to support effective teaching and assessment for learning. Support for under-achieving students is essential as is ensuring that all teachers see themselves as making a

contribution to children's numeracy. Parents too need to be involved. All these areas call for a whole-school commitment and cannot be left to a few teachers.

For students to become numerate, they must be given opportunities to practise and apply the mathematics they have learned. This should take place, not only in the mathematics classroom, but in other areas of the curriculum.

Collaboration between primary and secondary schools is important, especially across the transition years. Sharing of good practice with other schools helps to foster teacher professional development. Supporting all these directions is a key task for those who work at regional and network level.

Some of the reasons why a continued focus on numeracy is critical are suggested by the research in *Profiling high numeracy achievement* (Department of Education Science and Training 2004c):

- Many schools are yet to examine the relationship between mathematics and numeracy.
- Many primary and some secondary teachers lack confidence in teaching mathematics.
- Historically, mathematics in secondary schools has been a sorting and selecting device, resulting in mathematics being the least liked subject and disconnected from the experience of young people.
- As a school subject mathematics has tended to be dominated by transmission pedagogies which position learners as passive receivers of knowledge.

These same issues are outlined by the *Inquiry into the promotion of science and mathematics education* conducted by the Education and Training Committee of the Parliament of Victoria in 2005. The inquiry noted that effective mathematics learning, and hence the achievement of high levels of numeracy, depend on teacher quality. The inquiry received many submissions expressing concern about the level of mathematical knowledge and conceptual understanding among primary and some junior secondary teachers. Its recommendations included addressing the continued disengagement of many students from science and mathematics studies early in their secondary schooling; and increasing the level of appreciation among students, teachers and parents about the importance of continuing with mathematics and science studies for those seeking careers within vocational areas (Parliament of Victoria 2006).

Numeracy in Practice: Teaching, Learning and Using Mathematics is intended to bring to the attention of schools some findings of recent research in order to inform policy and practice at local level. It also draws attention to reports, resources and other materials provided by the Department to support school leaders and teachers of mathematics. Although many research studies are referred to in this publication, it is not intended to be an exhaustive treatment. Many of the issues discussed in this paper are discussed in more detail in the *Final Report of the National Numeracy Review* (COAG 2008).

2. Numeracy and mathematics

Numeracy is closely related to mathematics. Without a solid grounding in mathematical concepts and procedures, there can be *no* numeracy. On the other hand, knowledge of mathematical concepts and procedures *alone* is not enough to guarantee numeracy. What mathematics is taught and how it is taught has an important bearing on the development of young people's numeracy.

Looked at from another point of view, mathematics is a school subject. In all Victorian schools, classes are timetabled for the teaching of mathematics. This is not the case for numeracy. It may therefore be helpful to think of numeracy as a key *outcome* of how mathematics is taught and learned – something that is acquired and integrated with what students learn in their other school subjects, and in their wider experiences both in school and out of school.

The Ministerial Council for Education, Employment, Training and Youth Affairs (MCEETYA) in its 1997 *National Report on Schooling in Australia* stated:

Numeracy is the effective use of mathematics to meet the general demands of life at home, in paid work, and for participation in community and civic life.

The Australian Association of Mathematics Teachers (1998), in its report of the Numeracy Education Strategy Development Conference held in October 1997, specially addresses numeracy in the context of Australian schools:

In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; and grounded appreciation of context.

Whether one is talking about the school context or life outside of school, numeracy refers to the capacity to *use* mathematics to interpret information or to solve practical problems, and to apply mathematical knowledge appropriately in contexts where people have to use mathematical reasoning processes, choose mathematics that makes sense in the particular circumstances, make assumptions to resolve ambiguity and to judge what is reasonable (COAG 2008, p. xi).

In all references to numeracy in this paper, its mathematical underpinnings are assumed. 'Numeracy benchmarks', for example, well illustrate how these mathematical foundations are present or assumed. In many non-English speaking countries, numeracy is often called 'mathematical literacy'. Part of the reason for this is that the word 'numeracy' is hard to translate. Even in English, some people worry that the word 'numeracy' looks too much like 'number' and so runs the risk of acquiring a rather narrow meaning. One advantage of using the expression 'mathematical literacy' is to remind us that, like literacy in a language, we are not talking about a classroom teaching topic but a broad set of acquired behaviours and dispositions important for effective participation in society.

In contemporary Australia, school leavers and adults need mathematics for entry into the workplace and for future careers. In some careers, such as science and engineering, very high levels of

mathematical proficiency are required. However, regardless of their future careers, all young people need high levels of numeracy in order to participate effectively in Australian society and to be in control of their life and wellbeing. Unfortunately, for many students, mathematics learned at school is isolated knowledge. Numeracy is the capacity to bridge the gap between mathematics learned at school and the many contexts where it needs to be used in daily life.

To assist students to bridge that gap, several important things need to happen at school. In the first place, for students to become numerate, they must be given opportunities to practise and apply the mathematics they have learned, not only in the mathematics classroom but also in other areas of the curriculum. What is taught in school probably plays a greater role in the development of numeracy than in the case of literacy. Many out-of-school situations can help to foster language literacy, but some abstract mathematical ideas are not easy to acquire without the assistance of a teacher or parent who can assist young people to understand and use these ideas.

International understandings

The report by the UK Committee of Inquiry into the teaching of mathematics in schools (Cockcroft 1982) defined numeracy as the skills and dispositions needed by ordinary people in work and daily life. This definition reflects the shift of emphasis from earlier British definitions of 'numeracy' where there was a clear focus on skills with numbers, to definitions which include aspects of measurement and data handling and the application of mathematics skills to solving problems in particular contexts (Doig 2001).

Similarly, the British National Numeracy Project (Department for Education and Employment 2998) defined numeracy as:

more than knowing about numbers and number operations. It includes an ability and inclination to solve numerical problems including those involving money or measures. It also demands familiarity with the ways in which numerical information is gathered by counting and measuring, and is presented in graphs, charts and tables.

In the United States 'quantitative literacy' is a term more favoured than 'numeracy', although both are used. The National Council on Education and the Disciplines (ed. Steen 2001) describes elements of 'quantitative literacy' that include confidence with mathematics, cultural appreciation, interpreting data, logical thinking, making decisions, mathematics in context, number sense, practical skills, prerequisite knowledge and symbol sense.

Steen (2001) suggests that the terms 'quantitative literacy' and 'mathematical literacy' are used interchangeably, but often are used to signify important distinctions:

what is needed for life (quantitative) and what is needed for education (mathematics)

In a paper titled 'Mathematics for literacy' de Lange (2003) offers a model that relates a number of these terms, with mathematical literacy as the umbrella term encompassing the others. In de Lange's model, four content areas are used to organise mathematical literacy: Quantity; Space and Shape; Change and relationships; and Uncertainty. These are also the four content areas used by the OECD (2003) in its definition of mathematical literacy for the Programme for

International Student Assessment (PISA) for students in their last year of compulsory schooling where PISA defines 'mathematical literacy' as:

an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

In addition to these four content areas, PISA assesses eight mathematical competencies: *thinking and reasoning; argumentation; communication; modelling; problem posing and solving; representation; using symbolic, formal and technical language and operations; and use of aids and tools*. Taken together, the PISA content domains and competencies are similar to the mathematical dimensions of the Victorian Essential Learning Standards. Where PISA's definition of mathematical literacy comes even closer to the definition of numeracy used in this report is in its emphasis on 'engagement with mathematics in a variety of situations' (OECD 2003, p. 5). Four contexts are identified by PISA as being important contexts for using mathematics: *personal; educational or occupational; public; and scientific*. In Victorian schools, the emphasis given to each of these contexts may modify as students become older. For older students, a greater emphasis on occupational and scientific contexts may be expected to emerge. But a focus on numeracy as *using* mathematics cannot ignore any of these contexts. These contexts are also interrelated.

Steen (2001) returns us to a focus on how numeracy might best be addressed in schools, and underlines the importance of numeracy for success at school:

Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Rather, it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. Whereas mathematics is a well-established discipline, numeracy is necessarily inter-disciplinary. Like writing, numeracy must permeate the curriculum. When it does, also like writing, it will enhance students' understanding of all subjects and their capacity to lead informed lives.

Numeracy in the Australian mathematics curricula

'Numeracy' has been a priority for all Commonwealth, state and territory governments for more than a decade. This priority is reflected in the announcement of a National Plan for Literacy and Numeracy in March 1997 (MCEETYA 1997), and the Numeracy Education Strategy Development Conference in May 1997. Funding was provided to support a range of numeracy research projects.

The findings and implications of these projects are summarised in *Numeracy research and development initiative 2001–2004: An overview of numeracy projects* (Department of Education Science and Training 2005b). This report suggests that being numerate requires a blend of mathematical, contextual and strategic know-how.

In the various projects described in this paper and in the research studies referred to, the development of numerate behaviour at all

levels of school requires experiences along three dimensions of learning and using mathematics. These three types of 'know-how' form the first part of a Numeracy Framework developed by Willis and Hogan (cited in Department of Education Science and Training 2004b). These same three dimensions have also been highlighted by the National Numeracy Review (COAG 2008, pp. 11–12) as follows:

- *Mathematical*. The importance of building a strong foundation of mathematical concepts, skills and processes remains fundamental.
- *Strategic*. Giving clear attention to developing a toolkit of mathematical processes, appreciations and dispositions that enable young people to choose what mathematics to use in order to deal with familiar and unfamiliar situations.
- *Contextual*. Having experiences in applying mathematics to familiar and unfamiliar situations, students need to appreciate how different contexts and issues change the way mathematics is used and how the results are presented:

Example 1. Seventy students need to go by mini-bus to a sports event. Each mini-bus can take twenty students. How many buses will be needed? (Answer 1. Four mini-buses are needed. An answer of 3.5 mini-buses is not acceptable.)

Example 2. Twenty students collect \$70 for a community service project. What is the average amount collected by each student? (Answer 2. The average amount collected by each student is \$3.50.)

The second part of a Numeracy Framework developed by Willis and Hogan (cited in Department of Education, Science and Training 2004b) refers to the three roles taken by a student: the *fluent operator* (smooth and almost automatic use of routine mathematics in a familiar context), the *learner* (using mathematics to make sense of a new or unfamiliar situation) and the *critic* (knowing when and how mathematics can help and when it cannot).

Researching numeracy teaching approaches in primary schools (Department of Education, Science and Training 2004d), highlights the common features of the many numeracy projects undertaken in Australian states and territories over the past decade.

Firstly, they tend to refer to why numeracy is needed, for example: numeracy like literacy provides key enabling skills for individuals to participate in schooling. Furthermore, numeracy equips students for life beyond school (Department of Education, Training and Youth Affairs 2000).

Secondly, they variously refer to the ability to choose, use and communicate mathematical knowledge and skills relevant to context – numeracy involves abilities which include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles (Department of Employment, Education, Training and Youth Affairs 1998).

A third feature that these views have in common is their insistence that numeracy is 'more than number'.

These aspects of numeracy have been incorporated into mathematics curricula throughout Australia, both as applications within content strands – number, space, measurement, chance, data – and as part of process strands (often called 'working mathematically' or 'mathematical structure').

VELS and numeracy

In Victoria, the Victorian Essential Learning Standards (the Standards) describe five dimensions in the Mathematics domain: Number; Space; Measurement, Chance and Data; Structure; and Working Mathematically. All dimensions are equally important. In the early years – Prep to Year 4 – the foundations for effective numeracy are laid. Teaching and learning in subsequent years ensure that breadth and depth are added, with greater emphasis on contexts beyond school and ensuring development of numeracy that will support future work and further study.

Numeracy skills and dispositions are important in supporting studies in other domains in and across the three strands of the Standards: Physical, Personal and Social Learning, Discipline-based Learning and Interdisciplinary Learning. In turn, there are many contexts in domains across the strands that require the effective application of concepts, skills and processes from the five dimensions in the Mathematics domain, and contribute to strong numeracy development as part of student learning (Victorian Curriculum and Assessment Authority 2008).

The relationship between mathematics and numeracy is shown in the aims of the Mathematics domain which clearly specify that students are to:

- demonstrate useful mathematical and *numeracy* skills for successful general employment and functioning in society
- be empowered through knowledge of mathematics as *numerate* citizens, able to apply this knowledge critically in societal and political contexts
- *solve practical problems* with mathematics, especially industry and work-based problems
- *see mathematical connections* and be able to *apply* mathematical concepts, skills and processes in posing and solving mathematical problems
- be *confident* in their personal knowledge of mathematics, to feel able both to apply it, and to acquire new knowledge and skills when needed (VCAA 2008).

Summary

Across the world, while it is clear that not everyone uses the term 'numeracy', there is a strong consensus that all young people need to become competent and confident users of the mathematics they have been taught. Numeracy is best described as a key outcome of how mathematics is taught and learned – it bridges the gap between mathematics learned at school and the variety of contexts where it needs to be used in everyday life.

Numeracy is evident whenever mathematics skills and understandings are being used to analyse and solve a problem arising in real life, including school life. Mathematical knowledge that is seen as isolated from its applications, from other school subjects and from life outside school does not do justice to the important role that mathematics has in schooling and in the future life and work of all our students.

3. Australian performance in international studies

Australia participates in two international studies: Trends in International Mathematics and Science Study (TIMSS) which is conducted by the International Association for the Evaluation of Educational Achievement and the Programme for International Student Assessment (PISA) which is conducted by the Organisation for Economic Cooperation and Development (OECD).

TIMSS collects information on educational achievement data at Year 4 and Year 8 to provide information about trends in performance over time. The main goal of TIMSS is to assist countries to monitor and evaluate their mathematics and science teaching across time and across year levels (Thomson et al. 2008).

PISA, on the other hand, assesses mathematical literacy which is another way of referring to numeracy. PISA assessments focus on how well 15-year-old students, who in most countries are about to complete the compulsory years of schooling, are able to apply the mathematics they have learned in those years, to interpret and solve real-life problems.

PISA's domain of mathematical literacy is based on four broad content areas: Space and Shape; Change and Relationships; Quantity; and Uncertainty – once again driving home the point that mathematical literacy is much more extensive than mere number knowledge.

PISA provides information on profiles of student performance and engagement in reading, mathematics, and science; quality and equity in the performance of students and schools; and the school and system characteristics which influence performance.

In recent years, more than 40 countries participated in each of TIMSS and PISA. Participation in studies such as TIMSS and PISA allows educators to benchmark the performance of Australian students against the performance of students in other countries in science and mathematics. It enables assessment of Australian students' relative strengths and weaknesses and monitoring of achievement levels over time – nationally as well as from the perspectives of individual states and territories. It also provides information on how well Australian school systems are providing for the different educational needs of students from different backgrounds and in different locations (Stacey & Stephens 2008).

The TIMSS and PISA assessments are discussed in greater detail below.

TIMSS 2007

In Australia, a nationally representative sample of around 4000 Year 4 students from 229 primary schools and 4000 Year 8 students from 228 secondary schools took part in the TIMSS assessments in late 2006.

The results from TIMSS 2007 show that Australian Year 4 students have displayed some improvement in Mathematics achievement since 2003. But achievement levels of Australian students have remained static in Year 8 mathematics and Year 4 science and declined significantly in Year 8 science (ACER 2008a).

Internationally, Asian nations lead the way with the Russian Federation and Slovenia among those making big improvements since the last administration of TIMSS in 2003.

In Year 4, Australian students performed above the international TIMSS scale average in both Mathematics and Science. At Year 8, Australian students performed above the international scale average for Science and were on par with the international scale average for Mathematics.

The proportion of Australian students achieving results at the advanced international benchmarks set for TIMSS falls well short of the numbers of students in leading Asian nations reaching the top levels. For example, at Year 4 mathematics, nine per cent of Australian students achieved the advanced international benchmark compared to 41 per cent in Singapore and 40 per cent in Hong Kong. In reporting the findings, the Australian Council for Educational Research noted that while Australia's TIMSS results suggest an overall maintenance of our performance over time, this is in a context where other countries, including England and the United States, have made big improvements (ACER 2008a).

Thomson et al. (2008) report the following key findings from TIMSS 2007 from Australia's perspective:

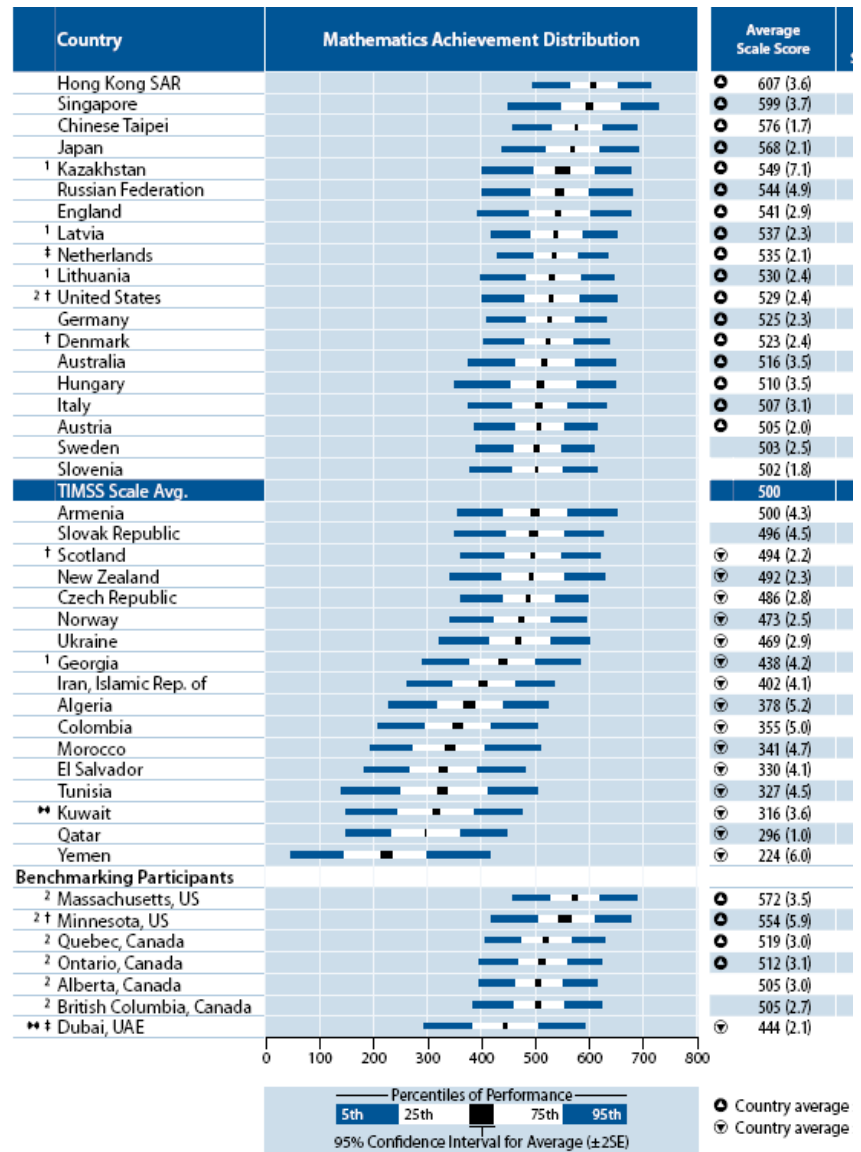
- Year 4 mathematics – Australian students' average scores in Year 4 mathematics have increased significantly by 17 points since 2003. In terms of relative position internationally, Australia was again outperformed by all of the Asian countries as well as England and the United States – a similar position to that obtained in 2003.
- There were some significant differences in Year 4 mathematics performance across the states. Students in New South Wales performed significantly better than students in all other states, except Victoria. Students in Victoria performed slightly below students in New South Wales, but significantly better than the remaining states, with the exception of the Australian Capital Territory, with which there was no statistically significant difference.
- Year 8 mathematics – the result for Australia is similar to 2003 but achievement scores have decreased since the first administration of TIMSS in 1995. Increases in scores achieved by students from England, the United States and Lithuania, in combination with a decrease in Australia's score, resulted in those countries significantly outperforming Australia in 2007. Overall, Australian students performed poorly in the areas of geometry and algebra.
- More than 10 per cent of Year 8 students in the Australian Capital Territory and New South Wales reached the advanced benchmark for mathematics achievement, but in all other states the proportion at this level was five per cent or less, with only one per cent of Year 8 students in the Northern Territory performing at this level. While this compares reasonably well with the international median, it is well short of the 45 per cent of students in Chinese Taipei that achieved at this level. The Australian Capital Territory also had the highest proportion of students achieving at least the high benchmark (34%), closely followed by New South Wales (27%) and Victoria (26%). The proportion of students achieving at least the low benchmark ranged between 84 and 93 per cent, in the Northern Territory and Victoria, respectively.
- Indigenous students – the results of an international study highlight that little has changed in regard to educational outcomes for

Indigenous students. At Year 4 the average score for Indigenous students in both mathematics and science was around 90 score points lower than that of their non-Indigenous counterparts. This gap has actually increased over time. Similar results were found at Year 8.

- Gender – in Australia, boys generally outperformed girls at both Mathematics and Science at each year level. This is in contrast to the international trend for girls to outperform boys.
- Trends – there was little movement in terms of achievement levels in the states. In Year 4 mathematics, New South Wales and Victoria had a significant increase in scores from 2003 to 2007, and New South Wales showed a sustained increase over the 12-year period from 1995. Scores in Western Australia were significantly higher in 2007 than 2003, while in Tasmania there was a slower but sustained growth over the 12-year period from 1995 resulting in a significant increase in scores from TIMSS 1995.

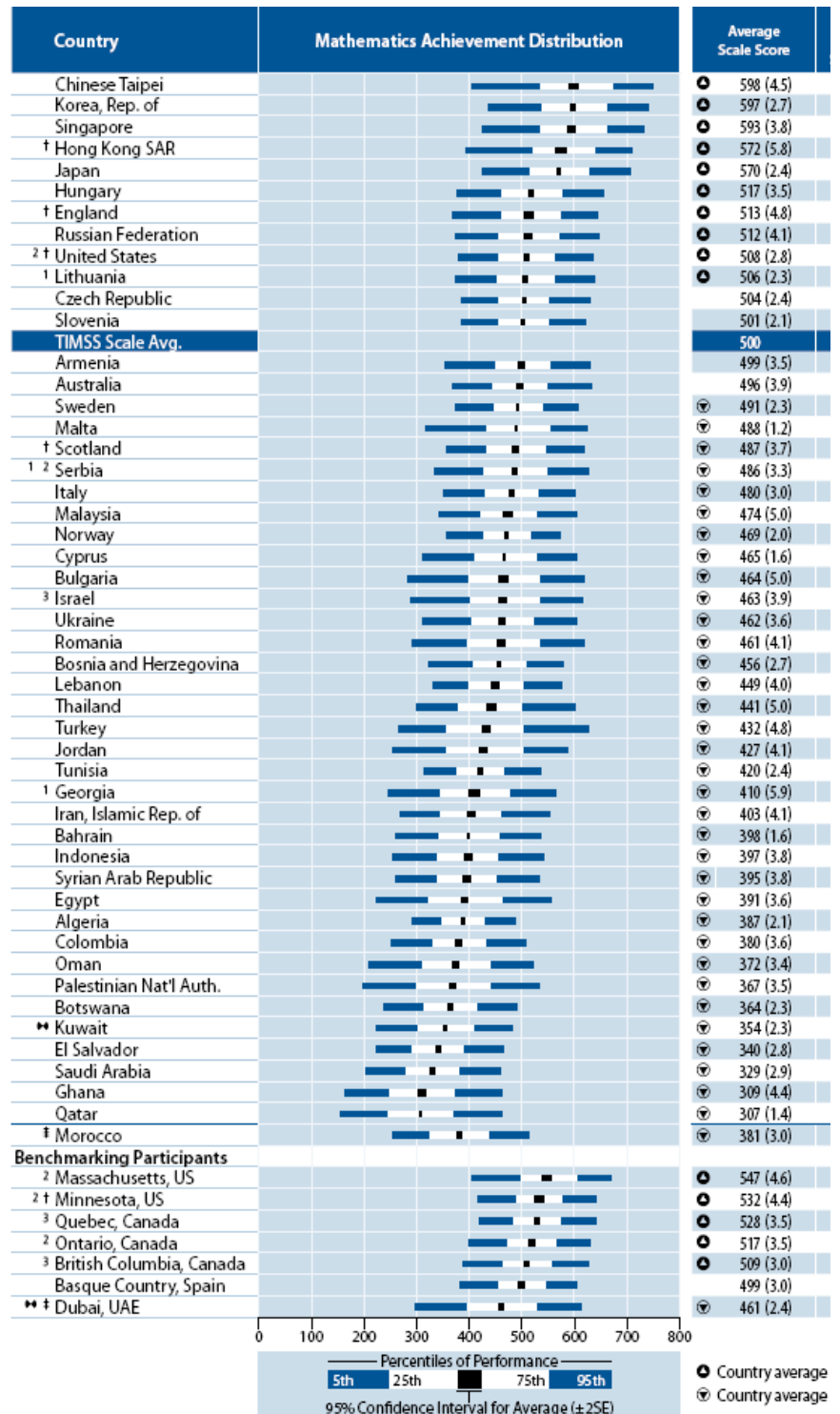
Figures 3.1 and 3.2 show the TIMSS 2007 distribution of mathematics scores for Year 4 and Year 8, respectively.

Figure 3.1: Distribution of TIMSS 2007 Mathematics Achievement at Year 4



Source: Thomson et al. 2008

Figure 3.2: Distribution of TIMSS 2007 Mathematics Achievement at Year 8



Source: Thomson et al. 2008

As the focus of this report is 'numeracy', only the information relevant to mathematics is reported here. More information on the contextual characteristics explored by TIMSS is available at:

<http://timss.bc.edu/TIMSS2007>.

PISA

In 2003, 41 countries participated in the PISA study. In all three cycles of PISA up to 2006, Australia's mathematical performance was above the OECD average. There was a small, but not statistically significant, decline in Australia's mean score from 2003 to 2006 (ACER 2008b).

The official reports of the results can be found at the ACER website: <http://www.acer.edu.au/ozpisa/kf_maths.html> and <<http://www.acer.edu.au/ozpisa/reports.html>>.

According to the summary report for Australia, Australia's results were above the OECD average in each of mathematical, scientific and reading literacy, as well as in problem solving, and in each of the mathematical literacy subscales.

In mathematical literacy four countries outperformed Australia in PISA 2003 – Hong Kong-China, Finland, Korea, and the Netherlands. In PISA 2000 only two countries performed better than Australia – Japan and Hong Kong-China.

Looking at achievement in terms of the proportions of students achieving above or below each benchmark level used in PISA assessments, a similar picture emerges in relation to the proportion of students achieving at the highest levels. PISA uses six benchmarks to look at how a country's performance is spread across the available range of performance. In PISA 2003, the highest scoring countries, Finland and Korea, had about 24 per cent of students in the highest two levels compared to 20 per cent of Australia students. In PISA 2006 the proportion of Australian students in these top two levels had dropped to 16 per cent. The proportion of Australian students failing to achieve what PISA defines as its 'baseline level of proficiency' is 13 per cent. While lower than the international mean, this is no cause for complacency.

Of particular concern is the fact that Indigenous students achieved a mean score equivalent to more than one full mathematical literacy proficiency level (or almost two and a half school years) below non-Indigenous students. There was also a large difference in achievement levels between students attending metropolitan schools and those attending schools in remote locations.

More information on the contextual characteristics explored by PISA is available at: <<http://www.pisa.oecd.org>>.

Lessons for Australia

Hollingsworth, Lokan and McCrae (2003) focus on making comparisons and commentary from an Australian perspective on the results of the earlier Third International Mathematics and Science Study (TIMSS) Video Study (ACER 2008c). International findings of the TIMSS Video Study 1999 of Year 8 mathematics teaching indicate that there is no single approach to successful teaching of mathematics. Hollingsworth et al. make the following observations:

- Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking.
- Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare.

- The Australian results expose ‘a syndrome of shallow teaching, where students are asked to follow procedures without reasons’, and that more than ‘shallow teaching’ is needed for students’ conceptual understanding and problem solving abilities to improve.

From their analysis of Australian Year 8 classrooms, Hollingsworth et al. (2003) note that:

- more than 75 per cent of the problems used by teachers were low in complexity (i.e. requiring relatively few steps to solve)
- the majority of the problems focused on carrying out of mathematical procedures
- only 25 per cent of the problems involved any connection with real-life situations, compared to 42 per cent in the Netherlands.

These observations point to areas for improvement in effective teaching. There is no evidence from the TIMSS 2002–03 study to suggest that the situation has significantly changed. The overall conclusion drawn by the study, according to Stacey and Stephens (2008), is that different methods of teaching can lead to high achievement, so that there is no ‘silver bullet’ to improve mathematical performance that will work across different countries. Japanese teachers developed lesson plans around a small number of rich problems, with deep cognitive content and careful attention to lesson objectives and mathematical connections. In Hong Kong, another high-achieving country, teaching was procedurally oriented, as it was in Australia, but compared with Australia, there were more complex problems, less repetition, and more emphasis on mathematical connections.

On many measures, Australian lessons were very similar to most of the other countries. There were, however, some differences between the way Australian teachers taught higher and lower ability classes. For example, lower-achieving classes had less algebra, more collaborative work and more interaction between the teacher and students. Stacey and McCrae (2003) also compared the Australian average Year 8 mathematics lesson with those of the high-achieving countries in the TIMSS 1999 Video Study. Stacey and McCrae also drew attention to ‘shallow teaching’ in a typical Australian middle school mathematics lesson, where the Australian data registered the highest national percentage of repetitive problems, the lowest percentage of problems which were ‘mathematically related’ (that is, where a problem would lead the student to extend particular method(s) used in a previous problem), and highest percentage of problems of low complexity.

In order to explore how mathematics is taught the Learner’s Perspective Study (Clarke et al. 2006a; 2006b) has teams of researchers in fifteen countries studying Year 8 classes using three video cameras and a sophisticated events categorisation (coding) procedure. The study recorded sequences of lessons by ‘competent teachers’ (as defined by the local community) and then analysed for a variety of purposes. Post-lesson video-stimulated interviews are also used to explore student beliefs and understandings.

Clarke (2006b) comments that:

One of the more intriguing outcomes of recent international comparative research is the diversity of classroom practice that characterises even those countries with similar levels of student achievement. Students in Japan, the Netherlands, and the Czech Republic have performed consistently well on

international tests of mathematics performance, yet the pictures that are emerging ... of mathematics classes in these countries are very different ... Such findings ... suggest that 'good practice' is a culturally determined entity (p. 11).

Improving numeracy outcomes in Victoria

In April 2007, Victoria released its literacy and numeracy plan (Department of Premier and Cabinet 2007), which sets out a ten-year vision for the Victorian and Commonwealth Governments to work together to:

- **value good teaching** and make sure Victoria has the best teachers possible in all our schools
- build a **culture of improvement** in schools
- direct resources to where they can most **make a difference**.

The plan acknowledges that while Australia's overall literacy and numeracy performance compares favourably internationally, urgent attention is needed to close the gap in literacy and numeracy achievement for students who are currently performing below the benchmarks. It is also necessary to do more to challenge and engage students – especially in maths and science – and to provide gifted and talented students with more opportunities to fulfil their potential (p.18).

Under the National Reform Agenda, COAG has agreed to explicitly measure progress towards achieving an increase in the proportion of young people meeting basic literacy and numeracy standards, and improving overall levels of achievement. The headline measures for measuring progress towards education and training outcomes for numeracy and literacy (for Years 3, 5, 7 and 9) are:

- the proportion of students achieving at or above the (minimum) benchmark standard
- the proportion of students achieving at or above the proficient standard.

The National Reform Agenda's focus on improving literacy and numeracy skills complements the *Melbourne Declaration on Educational Goals for Young Australians* (MCEETYA 2008) which states:

successful learners have the essential skills in literacy and numeracy and are creative and productive users of technology ... as a foundation for success in all learning areas.

If fully implemented, it is estimated that over 10 years Victoria's literacy and numeracy plan would have a significant impact on the COAG-agreed measures, namely:

- a 25 per cent reduction in the number of students who do not meet the minimum benchmark levels of achievement in literacy and numeracy testing for Years 3, 5, 7 (and Year 9 from 2008)
- a 10–15 per cent improvement in the number of students reaching a 'medium' benchmark 7 level of achievement in literacy and numeracy testing for Years 3, 5, 7 (and Year 9 from 2008) (DPC 2007).

The *Blueprint for Education and Early Childhood Development* (Department of Education and Early Childhood Development 2008a) articulates a specific goal for numeracy outcomes in Victorian schools:

In Year 5, Victorian students will have the basic numeracy skills to succeed in mathematics, and support will be in place for those who do not.

Support for schools will include teaching and learning coaches and a Differentiated Support Framework to provide guidance to teachers on responding to the range of student abilities and backgrounds they will find in any classroom.

An initial focus will be on accurate and timely assessment so students' progress can be monitored and support provided as soon as it is needed.

In deepening the implementation of the Victorian Essential Learning Standards, the focus will be on expanding the resources and opportunities for improving outcomes in mathematics and science, articulated in a mathematics and science strategy and a statement on Victoria's approach to literacy and numeracy.

The objectives of Energising Science and Mathematics Education 2009–13 are to:

- raise the science and mathematics achievement of every Victorian student;
- increase student interest in science and mathematics and encourage more students to pursue science and mathematics related careers to support Victoria's future prosperity; and
- expand knowledge of science and mathematics teachers and their capacity to engage students in contemporary science and mathematics programs.

Four key areas for action and funding within the strategy focus on:

- infrastructure
- workforce capacity
- curriculum resources
- strategic partnerships

The strategy is due for release in mid-2009.

The Literacy and Numeracy Statement will outline the key principles underpinning Victoria's approach to literacy and numeracy, including:

- Literacy and numeracy are foundational – achievement in these areas is critical to positive educational and labour market outcomes throughout life.
- Every teacher is a teacher of literacy and numeracy.
- Literacy and numeracy learning should be integrated across all areas and all stages of learning.
- Literacy and numeracy education should have real-world connections.

In addition to outlining areas of focus and possible improvement, the statement will reflect recent major developments in the educational environment in Victoria and Australia.

The Statement release date is still being determined.

Summary

Australia has been consistently above the international average in both TIMSS and PISA tests. Nevertheless, there are aspects of our performance that on closer inspection call for action. In particular, the performance of Indigenous students and those students living in remote and very remote locations is significantly lower than that of students in the rest of the nation.

Compared to those countries that are consistently high performing in these international assessments, Australian students are under represented in the top performing group and over represented in the lower performing groups, with a large proportion of students falling in the middle.

The National Reform Agenda and Victoria's plan to improve literacy and numeracy outcomes reflect a commitment by governments to keep improving performance in order to remain competitive.

The agenda for reform outlined in the 2008 Blueprint will focus on expanding the resources and opportunities for improving outcomes in mathematics and science, articulated in a mathematics and science strategy and a statement on Victoria's approach to literacy and numeracy.

The TIMSS Video Study (ACER 2008c), moreover, offers important insights into how mathematics is taught in typical classrooms in other countries. There are lessons to be learned from these countries that Australian schools might take to heart – in particular, more real-life connections, more mathematical connections and more high level questioning.

4. Australian approaches to numeracy

This section outlines several significant Australian numeracy research projects (and a related New Zealand project) which demonstrate the development of numeracy through effective mathematics teaching; the importance of school leadership; and the use of assessment to support learning and plan instruction.

In each of the research projects discussed here, teachers in participating schools were given extensive professional development. This was aimed to assist teachers to be able to identify learning goals that were clear and developmentally appropriate for students. Classroom assessment played a pivotal role in planning for teaching and fostering learning. Teachers were assisted to provide students with frequent exposure to high level mathematical problems as distinct from routine procedural tasks (from worksheets or textbooks). Tasks were seen by students as engaging and relevant. Opportunities were provided for students to learn from other students by discussing alternative solution processes and to explain their thinking.

Numeracy in the early years of school

The five major programs addressing numeracy in the early years of school are outlined in table 4.1. Four are in Australia, and one is a New Zealand adaptation of an Australian program.

Table 4.1: Numeracy programs in the early years of school

	Early Numeracy Research Project [ENRP] (Vic)	Numeracy in the Early Years: Project Good Start	Count Me In Too [CMIT] (NSW), also Early Numeracy Project [ENP] (NZ)	First Steps in Mathematics series (WA)	
Nature of program	Three-year research project to investigate effective numeracy teaching	Research project to identify effective numeracy programs	Numeracy program for early years of school	Series of Numeracy resource books based on extensive research	
Student groups	Prep to Year 2	Pre-school and Year 1 from all states and territories	Year 1 to Year 6	Primary school students	
Reference	Clarke et al. (2001), Clarke et al. (2006)	Thomson et al. (2005)	Wright et al. (2000), Wright et al. (2002), Thomas et al. (2002)	Department of Education and Training of Western Australia (2004)	
Key elements	Assessment/monitoring student progress	Developed by the project: Early Years Numeracy Interview*	Who am I? (preschool); I Can Do Maths (Year 1)	Developed by the program: Schedule of Early Numeracy	Ongoing evaluation by teacher
	Developmental Framework	Developed by the project: 'growth points' defined for strands in the Victorian mathematics curriculum	State or Territory curricula	Developed by the program: began with Number Framework, since extended to other strands	Developed by the program, matched to the WA curriculum framework: a Diagnostic Map for each strand showing Phases of Development
	Teacher professional development	A major focus of the project: teachers were co-researchers with the project team	Not a purpose of the project	A major focus of the program: ongoing information on teacher actions included as part of the program	PD on how to use the program

*now called Mathematics Online Interview

This focus on the early years emphasises the importance given to numeracy as a foundation skill. It is important to note that all five

programs listed above defined numeracy in ways that that are completely consistent with the five dimensions of the mathematical domain of the Victorian Essential Learning Standards – this was especially true of the Early Years Numeracy Research Project. The final report of Numeracy in the Early Years: Project Good Start (Thomson, Rowe, Underwood & Peck 2005) highlights two reasons given for focusing on numeracy in the early years:

1. The foundations of numeracy are laid in the experiences of children as they 'undergo unparalleled cognitive, social and emotional growth'.
2. A large number of children enter pre-schools and schools with already well-developed numeracy skills (p. 5).

Project Good Start analysed student results on standardised tests to identify effective numeracy programs in schools. The other research projects have several features in common:

- an assessment instrument for establishing initial student knowledge and understanding
- a framework describing stages of student development that enables student progress to be monitored
- an ongoing program of teacher professional development linked to other aspects of the program.

These early years studies have received a lot of attention from researchers. Tozer and Homes (2005) in their report on the NZ Early Numeracy Project, and (Bobis, Clarke et al. (2005) in their report on the NZ, NSW and Victorian projects, assert that:

Each program has been significant in rethinking what mathematics and how mathematics is taught to young children.

and

All three programs are based on a longer-term view of teacher growth and a view of teachers as reflective practitioners.

Wright, Martland and Stafford (2000) and Wright et al. (2002) describe a teaching approach and related teaching activities that have been developed in the Mathematics Recovery Programme in the UK to advance the numeracy levels of young children assessed at low attainers. Wright's Learning Framework in Number defined levels of children's understanding of number, counting, place value and addition and subtraction problems, and provided the basis for the developmental frameworks in number used in the Count Me In Too program in New South Wales and the New Zealand Numeracy Development Project.

In New Zealand, Young-Loveridge (2005) reports that the outcome of the Numeracy Development Project (the Early Numeracy Project) is that:

students who participated in the Numeracy Development Project in 2004 made significantly better progress on the Number Framework than would have been expected simply as a result of getting older. The advantage of being involved in the Numeracy Development Project was so great that it put younger students significantly ahead of slightly older peers who had not yet participated in the project.

Using the First Year of School Mathematics Interview (now called the Mathematics Online Interview) from the Early Numeracy Research Project, Clarke, Clarke and Cheeseman (2006) report that many children, on arrival at school, already clearly understood much of the mathematics curriculum for the first year of school. Clarke et al. propose:

As acknowledged by many ENRP Prep teachers, this means that expectations could be raised considerably in terms of what can be achieved in that first year.

Further:

The use of the interview, embedded within a professional development program, enhanced teacher knowledge and informed planning.

There are several clear reasons why these early years programs improve student achievement. Their classroom assessment component enabled teachers to map student progress over time. The programs usually incorporate a developmental framework which shows what the next steps are for students and helps teachers to interpret the results of assessments. These frameworks also give the teacher clarity as to what to teach and ideas for how to teach particular topics. All of these research projects have an ongoing professional development component which builds teacher expertise and confidence.

Reports detailing 'best practice' from the Early Numeracy Research Project related to specific topic areas include 'What does effective teaching of measurement look like? ENRP experiences' (McDonough 2003); *A shape is not defined by its shape: developing young children's geometric understanding* (Clarke 2004); and *Properties of shape* (Horne 2003). Each of these reports introduces developmental frameworks that use 'growth points' to define students' levels of understanding.

Both Clarke (2004) and Horne (2003) report that there was a significant increase in the growth of children's geometric understanding in trial schools. Horne states:

The ENRP (Early Numeracy Research Project) has shown in comparison of trial and reference school data that attention to geometry in the curriculum can make a difference in children's learning and they are capable of developing in geometry beyond the expectations of the last few decades.

Effective teachers in the trial schools seemed able to articulate mathematically focused, developmentally appropriate and engaging activities for their students, and engage them actively in interrogating and reflecting on those experiences (McDonough 2003). When teachers were asked if their teaching had changed as a result of the ENRP, Clarke (2004) reports that:

the use of 'growth points' to inform planning was the most common category mentioned.

Numeracy in the middle years

The Middle Years Numeracy Research Project: 5–9 (Siemon, Virgona & Corneille 2002) focuses on numeracy development in the middle years of schooling and reports on:

- the development of a strategic and coordinated approach and advice for schools about the learning and teaching of numeracy for students in Years 5–8+
- the trial and evaluation of the proposed approaches in selected Victorian schools; and what works and does not work in numeracy teaching.

A key challenge for mathematics learning, and for numeracy, in the middle years is the capacity to engage in multiplicative thinking. For the Middle Years Numeracy Research Project: 5–9, Siemon (2005) characterised multiplicative thinking by:

- a capacity to work flexibly and efficiently with an extended range of numbers (e.g. larger whole numbers, decimals, common fractions, ratio, and per cent)
- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion
- the means to communicate this effectively in a variety of ways (e.g. words, diagrams, symbolic expressions, and written algorithms).

In short, **multiplicative thinking** is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts (Siemon 2005). The development of multiplicative thinking was a major learning goal of this project. It marks an important step in children's mathematical thinking as they move beyond 'additive processes' – that is, increasing (or decreasing) by adding (or subtracting) fixed amounts – and begin to apply multiples, fractional parts, percentages, and proportions of given quantities to measure change. The capacity to engage in this kind of thinking is fundamental to numeracy in dealing with, for example, many contexts involving money, measurement, and space.

Rich assessment tasks involving these ideas and an extended activity were developed to provide pre- and post-program data on student knowledge and their ability to apply and communicate this knowledge in context. School action plans for the twenty selected trial schools were developed using the key design elements of the General Model for School Improvement (Hill & Crévola 1997).

To complement these learning goals, the Middle Years Numeracy Research Project: 5–9 developed a teaching and learning framework called Scaffolding Numeracy in the Middle Years. This framework is described in more detail in section 5: Effective numeracy teaching, and illustrated in table 5.1.

Among the many findings of this project are:

- all trial schools demonstrated an improvement in student numeracy performance
- improvements in numeracy outcomes were largely achieved as a consequence of a concerted focus on recognised 'best practice' in the teaching and learning of mathematics ... (but)
- consideration also needs to be given to how learning is organised and supported in the middle years of schooling ...
- the 'crowded curriculum' syndrome provides little space for connecting, generalising and conjecturing, and the primary focus on 'doing', as opposed to inquiry tends to generate passive learning and poor learning habits.

- in most Year 5 to Year 9 classes teachers can expect a range of up to seven school years in numeracy-related performance.
- a significant number of students in Years 5 to 9 appear to be experiencing difficulty in relation to some aspects of numeracy.

The further development of this research, the ARC Linkage project titled 'Scaffolding Numeracy in the Middle Years 2003–2006' (Siemon 2006) included the construction of a developmental framework, the Learning and Assessment Framework for Multiplicative Thinking.

Numeracy across the curriculum

The Numeracy Research and Development Initiative *Numeracy across the curriculum* (Department of Education Science and Training 2004b) focused on numeracy teaching across the primary school curriculum in sixteen Western Australian schools, utilising a Year 5 and a Year 7 class in eight schools.

The key findings from this project supported the emphasis given by Willis and Hogan (in DEST 2004b), on having contextual, strategic and mathematical components of numeracy mesh together. For students:

- The project found that familiarity with the *context* of a task matters. Students already familiar with a context were readily able to engage with and complete the tasks with mathematical demands but sometimes do this without necessarily being aware of 'doing mathematics'. They just 'know' what to do.
- *Strategic* skills are important such as knowing that mathematics might help, adapting mathematics to the context, knowing how accurate to be, and knowing if the result makes sense in context.
- The confidence of the student has an impact on their capacity to take up the roles of being numerate.
- It was generally true that whenever *mathematical* demands were made on students across the curriculum some students had trouble understanding the mathematics involved.
- It was found that working with others helps students clarify and solve problems. They are able to discuss and observe peers' strategies and this helps them organise their thinking. More than that, by working with each other within a learning area and by learning with the teacher, students also learn how things are done and how they are dealt with in the learning area – a community of practice.
- Transfer of knowledge from the demands of a numeracy task to other situations where similar numeracy issues can be explored doesn't just happen – there situations need to be created.

For teachers, an important component of 'upskilling' (i.e. professional growth and training) in this program was developing their capacity to recognise 'numeracy moments' in other areas of the curriculum. Such numeracy moments were identified in all areas of the curriculum. They had the following features (p. 81):

- Numerous mathematical demands are placed on students across the curriculum.
- These mathematical ideas and techniques come from every strand of the school mathematics learning area.
- The mathematics doesn't always look like the mathematics from school mathematics classrooms. Because the task is embedded in another context, the mathematical idea or technique that might be

required in the situation must firstly be identified, recognised and applied. It is not being done for its own sake.

- In their formal mathematics curriculum students may not yet have been exposed to the mathematics that arises across the curriculum.
- The students' contextual knowledge impacts greatly on their choosing and using of mathematical skills and indeed their need to use mathematics.
- This study focused on teaching across the curriculum in schools, where the same teacher teaches all (or most) of the curriculum areas to a class. The situation becomes more complex in secondary schools where each class has a range of specialist teachers.

Several of these national projects have led to the development of specific initiatives for all Victorian schools. The current Mathematics Online Interview, for example, was originally developed as part of the Early Numeracy Research Project (1999–2001). From the Middle Years Numeracy Research Project: 5–9, there is the Learning and Assessment Framework for Multiplicative Thinking. Resources developed from this project and available to schools include assessment materials, learning plans, and authentic tasks. Further examples of Victoria's investment in programs to enhance numeracy outcomes are detailed in the Resources section.

Summary

Both the Victorian Early Numeracy Research Project and the Middle Years Numeracy Research Project provided evidence-based recommendations about school organisation and structures that support achievement in student learning outcomes in mathematics. The student learning data collected showed a seven-year range of student achievement in many middle years classes. They also found that a whole-school approach to numeracy improvement, supported by effective leadership, was a key element in achieving improvement in numeracy outcomes.

Each of these research projects embodied high levels of collaboration between researchers, schools systems and participating schools. Individual schools are simply not in a position to carry out similar research on their own. In making use of what has been learned from these projects, individual schools have to decide what to draw on, how that fits in with their own priorities, and what outside support is needed to get under way.

Outside support – whether from the region or network – is likely to be most effective when individual schools set their own clear priorities for promoting numeracy, adopt a whole school approach to effective teaching, use classroom assessment to assist planning for instruction and to promote student learning. Schools need to make provision to support students who are identified as underachieving and others who may need to be challenged. Mathematics teaching and assessment need also to be linked to frameworks for reporting such as the Victorian Essential Learning Standards. These issues will be discussed in following sections.

5. Effective numeracy teaching

Teacher knowledge and classroom culture have been factors frequently researched as contributors to effective teaching. Project Good Start (Thomson et al. 2005) found that effective teachers:

- have high expectations of all students and set challenging tasks and goals appropriate for each student
- integrate their content knowledge and their teaching skills to make connections that engage student interest and maintain involvement
- monitor student progress using their knowledge of each student's current achievement and the next steps appropriate for them, and provide feedback to the student
- enjoy mathematics and take pleasure in students' enjoyment and success.

The key elements emerging from studies examining effective numeracy teaching practices are a clear focus on concepts and thinking, an emphasis on valuing children's strategies, and encouraging children to share their strategies and solutions (Thomson et al. 2005).

Teacher knowledge and scaffolding

The National Numeracy Review (COAG 2008) reports that 'there is increasing agreement that the mathematical content knowledge required for teaching is connected to the teaching of particular content ... and that *how* teachers hold knowledge may matter more than *how much* knowledge they hold' (p. 66). This is clear reference to the importance of (mathematical) pedagogical content knowledge for effective teaching, and hence the role that teacher professional development must play in developing that kind of specialist knowledge.

Shulman (1987) distinguished three different types of knowledge needed for teaching:

- *Content knowledge* – knowledge of the mathematics being taught, the amount and organisation of the subject matter per se in the teacher's mind.
- *Pedagogical knowledge* – knowledge of generic teaching strategies, such as questioning, grouping, planning, assessing, general factors that might impact learning.
- *Pedagogical content knowledge* – knowledge of the ways of representing and formulating the subject that makes it comprehensible to others, which includes knowledge of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them. For mathematics teaching and its relation to numeracy achievement, teachers' Mathematical Pedagogical Content Knowledge (MPCK) is cited as a key variable in many research studies.

Ma (1999) in her widely reported study of the differences between US and Chinese teachers pointed to four aspects of knowledge-for-teaching. These are:

- knowledge of basic mathematical ideas (i.e. the mathematical ideas that are pertinent to school mathematics)
- the ability to make connections between these ideas

- a capacity to create and use multiple representations of these ideas in teaching
- a deep knowledge of the curriculum continuum.

Her study found that while US teachers had generally taken more tertiary courses than their Chinese counterparts, they 'displayed less subject matter knowledge and (less) pedagogical content knowledge' (Ma 1999 cited in COAG 2008 p. 66). Her study underlines the importance of valuing the mathematical knowledge required for teaching in the primary and middle school years. This is not the same as a knowledge of advanced mathematics.

Teachers – at any level – who possess high-level mathematical knowledge have to work with children who clearly have more basic forms of that knowledge. A transmission model of teaching is not supported by research. Fennema and Romberg (1999) use the term 'cognitively guided instruction' that aims to understand and build on what the student is thinking. This places greater demands on knowledge-for-teaching since children's responses and strategies can take the lesson in many possible directions. The teacher's role is then to draw together those different directions with a clear focus on enhancing students' understanding.

Researching Numeracy Teaching Approaches in Primary Schools (Department of Education, Science & Training 2004d) identifies a range of interaction patterns or scaffolding practices undertaken by the teacher ('scaffolding' because of the support they provide the student in their learning process until the student is ready to 'stand on their own').

The scaffolding practices can be selected and used appropriate to purpose, for example, to explore/make explicit what is known, challenge/extend students' mathematical thinking, demonstrate the use of a mathematical instrument, or to assist students arrive at a key generalisation. In particular, they support teachers to make more informed decisions about how they will meet the learning needs of all students in the most appropriate way possible.

The twelve scaffolding practices that contribute to improved student learning outcomes are listed and described in table 5.1. When teachers used these scaffolding practices it had an effect on their own perceptions of what to teach and how to help students learn – that is, on their MPCK. As a result, there was a significant shift in what teachers perceived to be associated with effective mathematics teaching during the project. They changed from a predominant focus on activities, although these were still seen to be important, to recognition of the importance of teacher knowledge and the role of classroom culture (Department of Education, Science & Training 2004d).

Table 5.1: Scaffolding practices

Excavating drawing out, digging, uncovering what is known, making it transparent	Teacher systematically questions to find out what students know or to make the known explicit. Teacher explores children's understanding in a systematic way
Modelling demonstrating, directing, instructing, showing, telling, funnelling, naming,	Teacher shows students what to do and/or how to do it. Teacher instructs, explains, demonstrates, tells, offers behaviour for imitation

labelling, explaining	
Collaborating acting as an accomplice, co-learner/problem-solver, co-conspirator, negotiating	Teacher works interactively with students in-the-moment on a task to jointly achieve a solution. Teacher contributes ideas, tries things out, responds to suggestions of others, invites comments/opinions in what she/he is doing, accepts critique
Guiding cuing, prompting, hinting, navigating, shepherding, encouraging, nudging	Teacher observes, listens, monitors students as they work, asks questions designed to help them see connections, and/or articulate generalisations
Convince Me seeking explanation, justification, evidence; proving	Teacher actively seeks evidence, encourages students to be more specific. Teacher may act as if he/she doesn't understand what students are saying, encourages students to explain, to provide/obtain data
Noticing highlighting, drawing attention to, valuing, pointing to	Teacher draws students attention to particular feature without telling students what to see/notice (i.e. by careful questioning, rephrasing or gestures), encourages students to question their sensory experience
Focusing coaching, tutoring, mentoring, flagging, redirecting, re-voicing, filtering	Teacher focuses on a specific gap (i.e. a concept, skill or strategy) that students need to progress. Teacher maintains a joint collective focus and provides an opportunity for students to bridge the gap themselves
Probing clarifying, monitoring, checking	Teacher evaluates students understanding using a specific question/task designed to elicit a range of strategies, presses for clarification, identifies possible areas of need
Orienting setting the scene, contextualising, reminding, alerting, recalling	Teacher sets the scene, poses a problem, establishes a context, invokes relevant prior knowledge and experience, provides a rationale (not necessarily at the beginning of the lesson, but at the beginning of a new task/idea)
Reflecting/Reviewing sharing, reflecting, recounting, summarising, capturing, reinforcing, reflecting, rehearsing	Teacher orchestrates a recount of what was learnt, a sharing of ideas and strategies. This typically occurs during whole class share time at the end of a lesson where learning is made explicit, key strategies are articulated, valued and recorded
Extending challenging, spring boarding, linking, connecting	Teacher sets significant challenge, uses open-ended questions to explore extent of children's understanding, facilitate generalisations, provide a context for further learning

Apprenticing Inviting peer assistance, peer teaching, peer mentoring	Teacher provides opportunities for more learned peers to operate in a student-as- teacher capacity, endorses student/student interaction
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Some related overseas and Australian research

One part of the *Effective Teachers of Numeracy* study (Askew et al. 1997) considered the classroom organisation of effective teachers; the second part considered teachers' beliefs about teaching and mathematics. Effective teachers used a variety of classroom organisations, and while manifesting a range of different personal styles of teaching, it proved possible to discern three categories of approach to teaching:

- *Connectionist* teachers – who have beliefs and practices based on valuing children's methods, using children's understandings, and placing emphasis on making connections within mathematics.
- *Transmission* teachers – who have beliefs and practices based on the central role of teaching, and a view of mathematics as a collection of discrete skills, conventions and procedures to be taught and practised.
- *Discovery* teachers – who have beliefs based on the central role of learning, and a view of mathematics as being developed by children, particularly through interactions with concrete materials.

The connectionist teachers were found to be the most effective. However, when an attempt was made to replicate these findings in the Leverhulme Numeracy Research Programme, the results were less clear-cut (Brown 2000):

In the earlier study (Askew et al. 1997), there were clear-cut results in a small sample where relatively high mean gains were associated with teachers with *connectionist* orientations and relatively low mean gains with *transmission* and *discovery* orientations.

In the Leverhulme study we have found some of the same features as before in many of the teachers with high and low gains, but the picture is more complex, especially when trying to predict gains from features observed in lessons rather than teacher beliefs.

Nonetheless a *connectionist* orientation also appeared as a category for effective teachers in the Early Years Numeracy Project (ENRP). One of the aims of the ENRP (Clarke et al. 2001) was to gain insights into effective teachers and effective schools. Teachers whose students had shown 'high growth' were studied intensively through lesson observations, interviews and questionnaires. The distinguishing characteristics of effective teaching was classified into ten categories which can all be viewed as finer grained manifestation of pedagogical content knowledge:

- mathematical focus
- features of tasks
- materials, tools and representations
- adaptations/connections/links

- organisational style(s), teaching approaches
- learning community and classroom interaction
- expectations
- reflection
- assessment methods
- personal attributes of the teacher.

These categories are described in table 5.2.

Table 5.2: Effective teaching practices emerging from ENRP case studies.

Effective early numeracy teachers...	
Mathematical focus	<ul style="list-style-type: none"> • focus on important mathematical ideas • make the mathematical focus clear to the children
Features of tasks	<ul style="list-style-type: none"> • structure purposeful tasks that enable different possibilities, strategies and products to emerge • choose tasks that engage children and maintain involvement
Materials, tools and representations	<ul style="list-style-type: none"> • use a range of materials, representations and contexts for the same concept
Adaptions/connections/links	<ul style="list-style-type: none"> • use teachable moments as they occur • make connections to mathematical ideas from previous lessons or experiences
Organisational style(s), teaching approaches	<ul style="list-style-type: none"> • engage and focus children's mathematical thinking through an introductory, whole group activity • choose from a variety of individual and group structures and teacher roles within the major part of the lesson
Learning community and classroom interaction	<ul style="list-style-type: none"> • use a range of question types to probe and challenge children's thinking and reasoning • hold back from telling children everything • encourage children to explain their mathematical thinking/ideas • encourage children to listen and evaluate others' mathematical thinking/ideas, and help with methods and understanding • listen attentively to individual children • build on children's mathematical ideas and strategies
Expectations	<ul style="list-style-type: none"> • have high but realistic mathematical expectations of all children • promote and value effort, persistence and concentration

Reflection	<ul style="list-style-type: none"> • draw out key mathematical ideas during and/or towards the end of the lesson • after the lesson, reflect on children's responses and learning, together with activities and lesson content
Assessment methods	<ul style="list-style-type: none"> • collect data by observation and/or listening to children, taking notes as appropriate • use a variety of assessment methods • modify planning as a result of assessment
Personal attributes of the teacher	<ul style="list-style-type: none"> • believe that mathematics learning can and should be enjoyable • are confident in their own knowledge of mathematics at the level they are teaching • show pride and pleasure in individuals' success

Constructivist learning – an important clarification

Some may see a tension between the emphasis that research reported here places on the role of the teacher and what is often called 'constructivist' learning. However, the message from the research is clear: theories of learning and theories of teaching present complementary views. *How People Learn* (National Research Council 2000), a major analysis of current research into learning, states:

A common misconception regarding 'constructivist' theories of knowing (that existing knowledge is used to build new knowledge) is that teachers should never tell students anything directly, but, instead, should always allow them to construct knowledge for themselves. This perspective confuses a theory of pedagogy (teaching) with a theory of knowing. Constructivists assume that all knowledge is constructed from previous knowledge, irrespective of how one is taught ... there are times, usually after people have grappled with issues on their own, that 'teaching by telling' can work extremely well (p. 11).

Westwood (2004) supports this position:

It is now widely accepted that the most effective teaching approach combines important aspects of direct instruction together with the most meaningful and motivating components of student-centred learning.

Care needs to be taken with how the terms 'direct instruction' or 'direct teaching' are interpreted. It may be helpful to distinguish what the National Numeracy Review refers to positively as 'direct interactive teaching with high-level questioning' (COAG 2008, p. 35) from (direct) teaching that is based on 'teacher proof' materials or teaching that simply focuses on fast-paced, explicit instruction.

The differences between instructivist and constructivist approaches to the teaching of mathematics are outlined in table 5.3.

Table 5.3: Instructivist and constructivist approaches

	Instructivist	Constructivist
Also called	teacher-centred learning teacher-directed learning direct instruction*	student-centred learning self-directed learning discovery learning
Approach	Based on a transmission model: teacher = expert imparts knowledge to student = novice	Based on model of construction of knowledge by student/learner with teacher as facilitator
Characterised by	Reliance on textbooks Demonstration of correct method followed by student practice	Active construction of meaning through activities, discussion with other students Use of a variety of manipulatives
Example of a program	¹ Direct Instruction	² Cognitively Guided Instruction

*lower case 'direct instruction' is a generic term; capitalised Direct Instruction is a specific program

Instructional settings

Slavin and Lake (2007) and Slavin, Lake and Goff (2007) examined the relative effectiveness of different types of mathematics programs in primary and secondary schools. They classified the programs studied into three broad categories: those that were primarily textbook-driven; those that were primarily based around teaching and instructional interactions between teachers and students; and those that relied on computer-aided instruction. Across all school stages these researchers found that instructional programs were the most effective in raising mathematical achievement.

Some features of instructional programs that emerged as especially powerful were those that relied on cooperative and collaborative learning, especially in cases where students were performing below expectations or were considered otherwise at-risk. (See additional research studies cited in COAG 2008, p.35.)

Programs such as mastery learning (where students practice a given task until they master it) and direct instruction were found to have

¹ Originally developed at the University of Oregon, Direct Instruction is now a commercially published program from Science Research Associates. Originally intended as an accelerated learning program for at-risk students, but developed to be applicable for all students. Direct Instruction uses tightly scripted, fast-paced lessons. A placement test is used to group students according to achievement. Testing is repeated frequently, and students are regrouped as their achievement level changes.

² Cognitively Guided Instruction (CGI) was Developed by education researchers Thomas Carpenter, Elizabeth Fennema and others at the University of Wisconsin and is now published by Heinemann. CGI began as a professional development program. It emphasises the intuitive knowledge of mathematics that students bring to school, the processes they use to solve problems and the computational skills needed to solve problems. It requires a pedagogical shift by teachers to a student-centred approach to problem solving.

positive effects but were generally not as powerful. In some areas such as secondary school algebra (and also in some special education settings), direct instruction was shown to be very effective (Purdie & Ellis 2005).

Given the high degree of reliance on mathematics textbooks in teaching secondary mathematics, it is important to refer to the recent analyses of Australian mathematics textbooks by Stacey and Vincent (2008). Not only did they find wide variations in the extent to which textbooks made explicit mathematical connections. They found that textbook explanations are:

in general, very short with essential aspects of the reasoning unstated. Hence they are unlikely to stand alone, so students must rely on teachers to elaborate on the explanation provided. It is unlikely that all teachers can present these elaborations from the material provided ... This highlights the need for teachers' deep mathematical pedagogical content knowledge (p. 480).

To further clarify the role of instruction and to underline the importance of high-level mathematical interactions in instruction, Hiebert and Grouws (2007) identify two features of classroom teaching that are especially powerful for conceptual understanding and high mathematical attainment – *explicit attention to mathematical ideas, terms and procedures* (the same idea that is conveyed by *connectionist* teaching in the Askew et al. study) and cultivating the *engagement* of students.

Further confirmation of the importance of engagement in mathematics is provided by the Numeracy Research and Development Initiative Project, *What's making the difference?* (Department of Education, Science and Training 2005c). Researchers in this project investigated 65 NSW primary schools that were achieving outstanding outcomes in numeracy. Findings from this study pointed to the importance of students connecting (i.e. *engaging*) with the subject and with peers and teachers.

The New Zealand Ministry of Education *Best Evidence Synthesis* also relates *engagement* with students' having a positive and culturally appropriate mathematical identity (Anthony & Walshaw 2007). Mathematical identity refers to students seeing themselves as capable of learning/doing mathematics and seeing what they are doing as relevant and important.

Reference to the work of John Hattie and colleagues (Hattie 2003) provides a useful drawing together of features of instructional settings that promote high achievement and deep understanding. Hattie's work, which is based on many studies of effective teaching not limited only to mathematics teaching, found five major dimensions in the approach of expert teachers. Expert teachers can:

- identify essential representations of their subject
- guide learning through classroom interactions
- monitor learning and provide feedback
- attend to affective attributes, and
- influence student outcomes.

These five major dimensions lead to 16 'prototypic attributes of expertise'. The five dimensions and 16 attributes of expert teachers are outlined in table 5.4. Three attributes in particular were found to

provide almost all the differences between expert and experienced teachers:

Challenge: Expert teachers provide appropriate challenging tasks and goals for students.

Deep representation: Experts and experienced teachers do not differ in the amount of knowledge they have about curriculum matters or knowledge about teaching strategies. But experts do differ in how they organise and use this content knowledge. Experts possess knowledge that is more integrated, in that they combine new subject matter content knowledge with prior knowledge; can relate current lesson content to other subjects in the curriculum; and make lessons uniquely their own by changing, combining, and adding to them according to their students' needs and their own goals.

Monitoring and feedback: Expert teachers are more adept at monitoring student problems and assessing their level of understanding and progress, and they provide much more relevant, useful feedback.

It should be noted that Hattie's work did not focus on numeracy. However, a comparison of his findings with the categories that 'emerged' from Clarke's case studies, discussed above, show strong similarities.

Table 5.4: Characteristics of expert teachers

Dimension	Attribute
A. Can identify essential representations of their subject(s)	<p>A1. Expert teachers have deeper representations about teaching and learning.</p> <p>A2. Expert teachers adopt a problem-solving stance to their work.</p> <p>A3. Expert teachers can anticipate, plan, and improvise as required by the situation.</p> <p>A4. Expert teachers are better decision-makers and can identify what decisions are important and which are less important decisions.</p>
B. Guiding learning through classroom interactions	<p>B5 Expert teachers are proficient at creating an optimal classroom climate for learning.</p> <p>B6 Expert teachers have a multi-dimensional, complex perception of classroom situations.</p> <p>B7 Expert teachers are more context-dependent and have high situation cognition.</p>
C. Monitoring learning and provide feedback	<p>C8 Expert teachers are more adept at monitoring student problems and assessing their level of understanding and progress, and they provide much more relevant, useful feedback.</p>

	<p>C9 Expert teachers are more adept at developing and testing hypotheses about learning difficulties or instructional strategies.</p> <p>C10 Expert teachers are more automatic.</p>
D.Attending to affective attributes	<p>D11 Expert teachers have high respect for students.</p> <p>D12 Expert teachers are passionate about teaching and learning.</p>
E.Influencing student outcomes	<p>E13 Expert teachers engage students in learning and develop in their students' self-regulation, involvement in mastery learning, enhanced self-efficacy, and self-esteem as learners.</p> <p>E14 Expert teachers provide appropriate challenging tasks and goals for students.</p> <p>E15 Expert teachers have positive influences on students' achievement.</p> <p>E16 Expert teachers enhance surface and deep learning.</p>

The three darker shaded attributes are the three that contribute most to the difference between expert and experienced teachers (Hattie 2003).

Mixed ability groupings

One of the biggest challenges for teaching mathematics is providing for the wide range of confidence and mathematical understanding among students. As Clarke and Clarke (2008) point out, it is simply not possible to teach every student on an individual basis, and so teachers need to employ different strategies for dealing with the range of abilities in their classes. This wide spread of abilities is not restricted to mathematical ability, it may also involve reading ability and a number of other factors.

Some secondary schools, especially, deal with this challenge by assigning students to different classes according to mathematical 'ability' as determined by some test or according to students' previous performance. This kind of ability grouping is often called 'setting' or 'streaming'. In their extensive summary of the pros and cons of ability setting, Clarke and Clarke (2008) report that:

the research evidence is clear that generally any benefits which accrue from ability grouping are only to very high achievers, with a negative impact on average and low-attaining students.

They also raise a number of other important considerations which cast doubt on the practice of setting up more or less fixed groupings of students according to ability. On the other hand, groupings that are flexible and genuinely short-term are unlikely to lead to the negative outcomes referred to above. In particular, Clarke and Clarke (2008) strongly endorse practices such as:

pulling a small group aside for an extension task or to help a group having difficulty getting into a particular task or understanding a particular concept or skill.

A study by Lou et al. (1996) confirmed this view by showing in a large study that:

students working in small groups achieved significantly more than students not learning in small groups ... Low-ability and average-ability students learned significantly more in mixed-ability groups, while for high-ability students, group ability composition made no difference (cited in COAG 2008, p. 64).

The National Numeracy Review reports that while ability grouping is not endorsed by any official policies it is practised in many Australian schools. In a study of secondary schools in the UK, Wiliam and Bartholomew (2004) found that ability grouping does not raise average levels of achievement. Whatever benefits ability grouping may have for higher-achieving students, these are offset by poorer performances by lower-attaining students (cited in COAG 2008, p. 290).

However, the challenge remains of how teachers are to deal successfully with mixed abilities. Small group work is one of those strategies. Other writers have suggested the use of open-ended tasks. Effective use of these strategies assumes flexible groupings of students that are responsive to students' needs and based on frequent formative assessment. Students are not 'locked into' same ability groupings as they are with streaming or setting. Groupings may change depending on what mathematics is being taught.

In its submission to the National Numeracy Review, the Mathematics Education Research Group of Australasia (MERGA) made a clear distinction between allowing upper secondary students to choose courses and levels of study that are appropriate to their needs, future course options, interests and levels of attainment. However, in its submission, MERGA did not support the use of ability grouping in the primary and middle years of schooling. The MERGA submission made the telling point that countries that regularly outperform Australia on TIMSS and PISA make much less use of ability grouping and setting in primary and secondary classrooms.

Technology

The use of ICT in the teaching and learning of mathematics is rapidly developing. As digital technologies impact increasingly on life outside school and within school, we can expect them to have an impact on mathematics teaching and learning. Digital technologies already influence curriculum and assessment (e.g. Mathematics Online Interview) and will become more evident in shaping student learning. They cannot be excluded from mathematics classrooms, but they do need to be used sensibly. Just as 'digital literacies' do not displace the importance of developing competence in speaking and listening, there will be aspects of mathematics that will be technology-free, whereas in other areas technology has the potential to enhance the teaching and learning of mathematics.

As a feature of the changing environment of schools, it is clear that technology will always be ahead of research. Newer technologies have the potential for enhancing engagement of students in learning mathematics. The impact of ICT on motivation and engagement has been researched, but the rapidly changing nature of ICT itself makes

firm conclusions about its impact on achievement more difficult to reach.

White (cited in COAG 2008, p. 44) summarised the rapid changes that have occurred in the use of ICT by teachers of mathematics who:

used ICT initially for drill and practice, based on behaviourist theories and outcomes concentrating upon mastery of skills. Then the tool, tutor and tutee models ... became popular as these promoted higher order thinking and more student centred learning. This trend was supported by psychological theories about information processing ... cooperative learning and metacognition.

Digital technology first appeared in the form of simple hand-held calculators. There is still some disagreement about how extensively calculators should be used in the teaching and learning of mathematics. Hand-held technology is now capable of performing mathematically complex processes. The greatest impact of digital technology on schools is now likely to come from rapid developments in the World Wide Web.

World Wide Web

The use of the Web, especially through the internet has important potential for the teaching and learning of mathematics, especially through the means it provides to access information, such as real-world data, from a variety of sources that might be used, for example, in mathematical investigations or in investigations that are genuinely cross-curricular. Web 2.0 refers to changing uses of Web technology, including changes to Web design, that promote information sharing (not just accessing) and collaboration. Web 2.0 includes social networking, video sharing, and wikis and blogs where contributors are also authors. It is a generic name for web-based communities or for communities (e.g. schools and class groups) that communicate and collaborate on the Web. Potential uses of Web 2.0 for mathematics teaching and learning might include: discussion forums where students share ideas directly with other students or where this communication is mediated by the teacher; use of wikis for mathematical definitions, examples, problem solutions; sharing of images; and sharing applications. Use of this kind of Web technology means that mathematics teaching and learning is no longer restricted to a physical classroom or to particular times of the day (or night). It opens up possibilities for student-to-student communication and collaboration within and across classroom grouping and between schools. Unlike earlier forms of video-based communication through use of the internet, Web 2.0 allows communication that may be real-time or asynchronous.

Research on use of calculators

Data collected for TIMSS 1999 and for the TIMSS 1999 Video Study showed that calculators were used in 56 per cent of Year 8 lessons in Australia. In their analysis of the TIMSS data, Hollingsworth, Lokan & McCrae (2003) found no correlation between the use of calculators in mathematics classrooms in participating countries and the relative performance of those countries in the TIMSS assessments.

Recommendation 16 of the report by the United Kingdom's Numeracy Task Force (Department for Education and Employment 1998) relates to the use of calculators:

Calculators are best used in primary schools in the later years of Key Learning Stage 2, and not be used as a prop for simple

arithmetic. Teachers should teach pupils how to use them constructively and efficiently.

This supports the UK National Numeracy Strategy approach for an emphasis on mental calculation.

The judicious and careful use of calculators has been seen as a positive influence on the teaching of mathematics in Australian schools over the past twenty years. The Calculators in Primary Mathematics project (see Groves 1995) involved 80 teachers and 1000 students who took part in this 1992–94 study. This study showed how facilitating access to hand-held computational calculators in the early years led to significant and profound contribution to understanding, skill and performance. These achievements included success at mental computational tasks (see COAG 2008, p. 44).

Other projects, such as the Calculator-Aware Number project in England in the late 1980s, also point to pedagogical benefits of harnessing computational calculators in the primary mathematics classroom. In many primary schools, such calculators 'are used only for checking already-completed work or for special calculator activities' (Groves, Mousley & Forgasz 2006, p. 94).

Research elsewhere, for example Groves and Stacey (1998), has shown benefits in mathematical understanding for children using calculators at an early age.

Computers

Research on the impact of incorporating computers on student learning outcomes remains inconclusive. It is also singularly difficult to measure. Combined with uneven and sometimes low usage rates, recent Australian and overseas research has focused attention on how computers are actually used in classrooms. Ertmer's (2005) study reported the prevalence of computer use for relatively low-level tasks such as word-processing and internet searches and saw little evidence of their being used for high-level mathematical tasks. Zevenbergen and Lerman (2006) drew on data from Victoria and Queensland schools to argue that among the likely causes of the problem alluded to by Ertmer's study were teachers' own confidence and skills in dealing effectively with computers and ICT. This finding has clear implications for continuing teacher professional development.

Despite many claims about what computers might contribute to effective mathematics teaching, a key challenge is making sure that computing technology (and other forms of ICT) actually gets used. There are several barriers to effective use. These may in part be institutional, where some schools are unable to buy new technology or maintain or update the technology that they already have. Just as clearly, teachers themselves can either be promoters or barriers to effective use. The National Numeracy Review Report (COAG 2008, p. 47) identifies several key factors affecting teachers' use of technology. These include access to appropriate professional development, teacher's beliefs about the nature of mathematics and how it should be taught, and teacher beliefs about gender issues.

In regard to gender, Forgasz (2006), in a study involving 111 secondary school teachers of mathematics, reported that many teachers believed that boys and girls work differently with computers. Further, these teachers reported that girls tended to display less confidence, less competency and less interest in working with computers than boys in the comparable year group. These gender equity issues surrounding access to and use of ICT need further

examination. The ability of ICT to engage the interests of boys is referred to below (see Halford 2007). It cannot, however, be at the expense of other students.

Other technologies for teacher and student use

A report by the British Educational Communications and Technology Agency (BECTA 2006) provides information on the early progress of the strategy for technology use in England, *Harnessing Technology: Transforming learning and children's services* (British Educational Communications and Technology Agency 2006). The BECTA report provides information on the number of desktop and laptop computers, data-loggers and interactive whiteboards in use in schools, discusses developments in technology use in schools and reports on progress, issues and challenges facing the use of technology in schools. The Learning and Teaching section of the BECTA site has links to both primary and secondary mathematics support material.

Technology is increasingly being used in classrooms to support mathematics understanding. Advances in computer and calculator technology, and research into their use, can assist students' developing an understanding of mathematics concepts. For example, *The Teaching and Learning about Decimals* CD-ROM (Steinle, Stacey & Chambers 2002) provides a research-based resource for the effective teaching of decimals.

Oldknow (2005) reports on information and communication technology developments in the United Kingdom, including the mathematics pack *Embedding ICT @ Secondary Key Stage 3 Mathematics* (Department for Education and Skills 2005), an ICT resource available free to schools, that includes exemplar lesson materials as well as information on using interactive whiteboards. Oldknow also provides curriculum materials that use technology to link mathematics to the real world (<http://adrianoldknow.org.uk/>).

Interactive whiteboards

Following a large-scale evaluation of the impact of interactive whiteboards (IWBs) for the UK's Primary National Strategy's 'Embedding ICT' pilot project, Smith, Hardman and Higgins (2006) examined the impact of IWBs on teacher-pupil interaction in literacy and mathematics lessons. They reported that lessons using IWBs had significantly more open questions, answers from pupils and evaluation. However, in questioning the effectiveness of the in-service programs that have accompanied the introduction of the interactive whiteboards, Smith et al. suggest:

While it could be argued that the IWB is a useful presentation tool to have in the classroom, the findings suggest that such technology by itself will not bring about fundamental change in the traditional patterns in whole class teaching.

The research literature has yet to demonstrate the direction that teachers need to move to ensure that the proven changes the IWB can bring about in classroom discourse and pedagogy are translated into similar and positive changes in learning (Higgins, Beauchamp & Miller 2007).

Online resources

Multimedia software, and in particular, the online *Maths300* (Curriculum Corporation n.d.) resource developed from the successful Mathematics Curriculum and Teaching Program, are increasingly being used in school mathematics programs.

The University of Melbourne's RITEMATHS project (Stacey, Stillman et al. 2006) aims to create and implement a curriculum that uses technology to link mathematics to the real world for the middle secondary years and uses software-enabling analysis of digital photos and video.

The BECTA Report (2006) envisages:

though still representing pockets of leading-edge practice, there are clear signs of the development of effective use of learning platforms to support learning, teaching and management in a coherent, effective and efficient way. This is a key point at which 'best practice' to support educational processes in these areas can be established. Learning from early adopters is critical at this stage, and should be prioritised at all levels in the system.

The report concludes:

It is now becoming the norm, for example, that teachers prepare lessons using online resources. Use of display technologies is also becoming the norm. Thus we can see a noticeable embedding of technology into everyday learning and teaching practice.

Making effective use of technology – especially IWBs

The UK Advisory Committee on Mathematics Education (2006) reflects on the use of IWBs and recommends that the use of IWBs alone is not likely to improve learning and teaching:

Teachers need to be encouraged by their school to reflect on the impact that IWBs have on their own pupils' learning of mathematics and consider this in relation to the use of other teaching strategies.

The opportunity to improve the quality of learning and teaching through the provision of innovative, technology-based educational programs and services can be enhanced through linking research initiatives that aim to create and implement a curriculum that uses technology to link mathematics to the real world and by increasing access to technology, in particular, to IWBs (Stacey, Stillman et al. 2006).

Recent editions of *Teacher* magazine (March 2007 & August 2007) have provided several case studies of use of IWBs and other technology in the classroom. Among these studies, Halford (2007) reports on the challenges of introducing IWBs. While he recognises their potential:

IWBs provide an extraordinary opportunity to expand teachers' ability to engage students, particularly boys.

Halford also reports their challenges:

The cost of professional learning ... is very high and ongoing. Researching suitable internet sites ... has proved to be time-consuming. Troubleshooting and down time with the equipment has made demands on staff. The archiving of sites has emerged as a significant issue for us.

Halford also endorses The Learning Federation (n.d.) learning objects as an important and useful resource. In another study, Brown (2007) reports on interschool interactions via email/PowerPoint in WA between a remote Indigenous school and a Perth school, and between schools in Sydney and France. The Sydney/France exchange, *Beyond Borders*, began with a language focus, but has now expanded to

include maths, SOSE, visual arts, and to using voicemail and live audio-conferencing.

The successful integration of ICT is also discussed by Raiti (2007), who argues that a change of culture is needed:

Making sure that there are sufficient computers in your school won't necessarily encourage your teachers to use new technology. It's your school's learning culture which is more likely to determine whether desired improvements to teaching and learning can be effected through the integration of ICT.

The Victorian Department of Education and Early Childhood Development recognises that digital learning resources (DLRs) play a central role in integrating ICT into the curriculum, developing innovative practices and transforming the way students learn. In partnership with Intel®, the digiLearn online portal provides all Victorian government teachers with access to educational DLRs, including 'The Learning Federation' learning objects, to support students in the areas of Maths and Science for VELS Level 4 to 6. DigiLearn can be found at: <<http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm>>.

The eLearning and ICT Showcase (<http://epotential.education.vic.gov.au/showcase/>) provides examples of the potential of ICT to enable powerful teaching and learning in mathematics. The showcase also features how electronic whiteboards can be effectively used in teaching.

Local programs such as RITEMATHS (http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS/general_access/project_home/Outline.shtml) create and implement a curriculum that uses technology to link mathematics to the real world for the middle secondary years. These programs include 'real-world interfaces' which schools are now beginning to use. These include software enabling analysis of digital photos and video, dynamic control of simulations, and data logging devices. They also include 'Maths analysis tools' (i.e. spreadsheets, function graphing software, and computer algebra systems (CAS), on both calculators and computers) which work with abstract mathematical representations.

Effective school contexts for numeracy teaching

The report *School performance in Australia: results from analyses of school effectiveness* (Lamb, Rumberger et al. 2004), concludes that there is a range of factors that contribute to effective teaching and, in particular, effective numeracy teaching. In the analyses of achievement the following factors were significantly related to student achievement:

aspects of classroom practice and quality of teachers reflected in teaching styles and levels of satisfaction with teaching

and

the academic climate schools create reflected in the behaviour of students, broad aspiration levels, student views on teachers and school and engagement in school life.

The report further suggests that:

Schools that do well in achievement terms adopt policies facilitating student engagement, through provision of programs, extra-curricular programs and student support.

and also that:

Targeting improvements will need to take account of the interplay between these factors including resources, classroom practices, programs and teachers.

One of the Numeracy Research and Development Initiative projects was What's making the difference? (Department of Education, Science and Training 2005c). The report of its findings states:

In the first year, outstanding numeracy practices, strategies, programs and policies in 25 high-performing schools across NSW were identified on the basis of quantitative data analysis ... During the second year an additional 20 case studies were undertaken at other primary schools producing outstanding outcomes in student numeracy based on qualitative data analysis. While the two sets of case studies were identified using different criteria ... the results of the analysis revealed a high degree of consistency in the identified factors and strategies.

This research identified 'three interacting, but discrete, contexts' – within the classroom, throughout the school and beyond the school.

The main factors that were found to be occurring and that seemed to be 'making the difference' were analysed within these three contexts and practical strategies enabling these factors were derived from careful consideration of the practices of the case study schools.

The three contexts, the factors and their associated strategies are shown in table 5.5.

Table 5.5: Identified factors and strategies in 'high-performing schools'

Within the classroom

Factors	Strategies
Language as a focus for learning	1.1 Classroom organisation that lets learners use talk to scaffold learning 1.2 Effective scaffolding in mathematics classrooms, where a learner works with a peer of higher skill development level in a group or in a pair, or with a parent or teacher, on a learning task that involves the manipulation of resources or a problem-solving activity. 1.3 Expectations that learners will use oral and written language to explain strategies and justify conclusions 1.4 Questioning by both teachers and learners to establish, consolidate, extend, reinforce and reflect on concepts, skills and applications 1.5 Strong links between literacy and numeracy

Assessment to identify and accommodate difference	<p>2.1 Assessment to identify and monitor individual abilities</p> <p>2.2 Structures for managing different abilities through grouping or individualising learning</p> <p>2.3 Culturally aware pedagogy to build on cultural knowledge</p>
Purposeful pedagogy	<p>3.1 Identifying specific outcomes as the lesson focus, linking indicators to outcomes, structuring explicit teaching steps to achieve outcomes within a lesson structure that is predictable, with an introduction, activity focus and conclusion</p> <p>3.2 Resources that allow learners to manipulate concrete materials to construct their own understandings and use these resources in the application of concepts and skills</p> <p>3.3 Opportunities to make basic facts and skills automatic through drill and practice activities that are fun and usually implemented as an introduction to a lesson</p> <p>3.4 Classroom management that is appropriate to the context of the school and consistent with school policy, involving a strong (but sometimes discreet) teacher presence</p> <p>3.5 Strengthening teachers' mathematical knowledge and pedagogical strategies through professional development</p> <p>3.6 Opportunities for teachers to work collaboratively in planning and teaching to support innovation, sharing of ideas and resources</p>

Throughout the school

Factors	Strategies
A school commitment to numeracy	4.1 Leadership to unify and direct 4.2 Collaborative development of a document to direct mathematics 4.3 School organisation to support mathematics through resourcing, timetabling and class organisation 4.4 Structures to cultivate collaboration between staff to facilitate communication and a cohesive sense of purpose
School policies that support numeracy	5.1 Identification of areas that need to be addressed in mathematical teaching and learning through instruments such as parent and teacher surveys 5.2 Development, consistent implementation and school-wide monitoring of policies, particularly welfare policies
Specialised programs to support numeracy	6.1 Introduction of contextually and systemically appropriate structured programs, particularly the Count Me In Too (CMIT) programs (including CMIT Indigenous program, Counting On, Counting Into Measurement, Counting Into Space) 6.2 Special, school-wide programs to support learning in mathematics

Beyond the school

Factors	Strategies
A shared vision	7.1 Parent and community consultation in developing the school's vision 7.2 Developing clear goals 7.3 Programs to support parents' understanding of mathematics policies and practices
Communicating about learning	8.1 Formal reporting structures that provide opportunities for discussion 8.2 Portfolios that provide a comprehensive record of achievement and may include work samples, test results and reporting of outcomes 8.3 Informal reporting structures that enable parents and teachers to be informed of immediate issues 8.4 Homework as another way of reporting student progress
Mathematics at home	9.1 Providing parents with the knowledge and skills to support maths at home 9.2 Staff conduct workshops that relate to new initiatives in numeracy 9.3 Homework that invites the participation of the family rather than excluding it

These strategies were then applied in ten trial schools identified 'as having school numeracy outcomes at or below the state average':

In 2002 ten of the 19 cohorts from eight trialling schools demonstrated greater than expected growth when compared to their counterparts in reference schools. In 2003 this increased to 12 out of 19 cohorts. Given that the trialling schools had been selected because of low performance in numeracy and reference schools selected because of their performance at or slightly above the average, the gains in numeracy made by trialling schools were significant (DEST 2005c).

Teacher professional development

Ma (1999) in her widely reported study of the differences between US and Chinese teachers pointed to four aspects of knowledge-for-teaching. These are:

- knowledge of basic mathematical ideas (i.e. the mathematical ideas that are pertinent to school mathematics)
- the ability to make connections between these ideas
- a capacity to create and use multiple representations of these ideas in teaching
- a deep knowledge of the curriculum continuum.

Her study found that while US teachers had generally taken more tertiary courses than their Chinese counterparts, they 'displayed less subject matter knowledge and (less) pedagogical content knowledge' (Ma 1999 cited in COAG 2008 p. 66). Her study underlines the importance of valuing the mathematical knowledge required for teaching in the primary and middle school years. This is not the same as a knowledge of advanced mathematics.

Teachers – at any level – who possess high-level mathematical knowledge have to work with children who clearly have more basic forms of that knowledge. A transmission model of teaching is not supported by research. Fennema and Romberg (1999) use the term 'cognitively guided instruction' that aims to understand and build on what the student is thinking. This places greater demands on knowledge-for-teaching since children's responses and strategies can take the lesson in many possible directions. The teacher's role is then to draw together those different directions with a clear focus on enhancing students' understanding.

The use of research-based frameworks and assessment strategies by teacher professional development programs, such as the Early Numeracy Research Project and Count Me in Too, have demonstrated important advances in children's learning. These and other programs discussed in this report support the central tenets of the US Cognitively Guided Instruction Project which state that:

knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. One major way to improve mathematics instruction and learning is to help teachers to understand the mathematical thought processes of their students (Fennema et al. 1996, p. 432)

The National Numeracy Review (COAG 2008) identifies the following features that appear to be shared across a number of successful research-based teacher development programs in Australia (these programs have been cited earlier in this report). The programs:

- focus on involving whole schools (or at least all teachers at the relevant grade levels)
- develop and use research-based frameworks (e.g. in Count Me In Too (NSW), First Steps (WA), Early Numeracy Research Project (VIC))
- are based on partnerships between schools, systems, and universities
- recognise the importance of school leadership in effective programs
- in assessment, focus on understanding individual student thinking in relevant mathematical domains
- often involve one-to-one interviews, as well as typical learning trajectories for these (recognising that such trajectories will not apply for all students)
- have a major focus on enhanced pedagogical content knowledge
- make a clear link to classroom practice, with opportunities for peer or other expert support within classrooms
- include strategies for addressing the needs of low-attaining students
- have a clear focus on ongoing reflective professional development (see COAG 2008, p. 95).

Professional teaching standards for teachers of mathematics

Working with groups of primary and secondary teachers throughout Australia, the Australian Association of Mathematics Teachers (AAMT) identified three domains that can be used both to identify accomplished teachers of mathematics and inform continuing teacher development (AAMT 2006).

Professional knowledge

Excellent teachers of mathematics possess a strong knowledge base in all aspects of their professional work including their decision-making, planning, and interactions. This includes knowledge of students, how mathematics is learned, what affects students' opportunities to learn mathematics and how the learning of mathematics can be enhanced. It also includes sound knowledge, training, and appreciation of mathematics appropriate to the grade level and/or mathematics subjects they teach.

Professional attributes

Excellent teachers of mathematics are committed and enthusiastic professionals who continue to extend their knowledge of both mathematics and student learning. They work creatively and constructively within a range of 'communities' inside and beyond the school and set high, achievable goals for themselves and their students. These teachers exhibit personal approaches characterised by caring and respect for others.

Professional practice

Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective, of the students they teach. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, the lessons they plan, their uses of

technologies and other resources, their teaching practices, and the ways in which they assess and report on student learning.

These standards can inform and assist sector and regional authorities, individuals and groups of teachers to identify needs, set directions and targets, and establish 'distance travelled' in relation to professional learning (Bishop, Clarke & Morony 2006). They also have a capacity to inform the development of national numeracy teaching standards.

Summary

From the early years of schooling students need to be provided with higher-level problems and questions which help them make connections between key concepts and procedures, rather than instruction that is focused on routine procedural tasks. Students need to see the mathematics they are doing as important and relevant and themselves as capable of thinking and working mathematically.

Working collaboratively in mixed ability groupings is generally to be encouraged with opportunities for students to support one another and to share explanations. Serious questions remain about the practice of more or less fixed ability groupings, either in the same class or placing students in different classes according to ability. This practice is not supported by research. On the contrary, it appears to contribute to negative attitudes and lowered attainment for less well-achieving students and to yield limited benefits for others.

ICT in its various forms has, and will continue to have, an important place in the teaching and learning of mathematics. New forms of Web-based technology have the potential to create new 'spaces' for learning and communication in all subjects. In particular, they have the potential to engage students as active contributors in their own learning, and to re-define the times and spaces when and where learning occurs.

Continued advances in technology itself and changed contexts in its use in schools create problems for longitudinal research studies of effectiveness that can confidently be ascribed to the use of technology itself. However, there are many formal and informal measures of student engagement, motivation and achievement that can be used to monitor the use of technology.

There is evidence of gender differences in the way technology is used. These need to be monitored carefully at classroom and school levels. Access by teachers to appropriate professional development in the effective use of technology – especially in promoting higher-level mathematical thinking – is also important.

Schools that demonstrate high performance in the numeracy domain develop a shared vision with goals that:

- take a whole-school approach to improving numeracy outcomes
- establish policies and structures that encourage collaboration among teachers
- introduce professional development programs to support teachers in becoming more effective
- involve parents and the wider community in developing and supporting the new numeracy initiatives.

6. Differentiating support to improve student learning

The Middle Years Numeracy Research Project (Siemon, Virgona & Corneille 2002) reported a seven-year range of student achievement in many middle years classes, and similar spreads of achievement and understanding can be found at other year levels. Coupled with the likelihood of having students from a variety of cultural or socio-economic backgrounds, with a wide range of language skills and with a variety of learning problems or special needs, this provides a major challenge for teachers to achieve increased growth for all. A whole-class approach to learning may not enable low-achieving children to fully participate and benefit from instruction. Several ways of providing for underperforming students are discussed here. Some involve out-of-class provision, others involve in-class provision. Each will be discussed.

Expressing general support for in-class provision, Westwood (2004) lists the following 'standard' adaptations that teachers might consider to ensure that more students experience success:

- make greater use of visual aids, concrete materials and practical demonstrations
- simplify and restate instructions for some students
- set shorter-term objectives for certain students
- ask questions of different degrees of complexity, ensuring that less-able students are asked many questions they can answer
- select or make alternative curriculum resource materials, including using different textbooks within the same classroom
- monitor the progress of some students more closely during the lesson, and reteach where necessary
- set students different forms of homework
- encourage peer assistance and peer tutoring
- make regular and appropriate use of group work, enabling students of different abilities and aptitudes to collaborate.

Westwood warns that another adaptation often suggested, to reduce and simplify the curriculum content and to slow the pace of instruction 'usually guarantees that a student will fall even more rapidly behind'. He recommends:

doing everything possible to keep the gap between higher and lower achievers to a minimum, and ... this is best accomplished through delivering a common curriculum using mainly high-quality, interactive, whole-class teaching

These issues will be elaborated further in following sections dealing with specific programs of in-class support (inclusive teaching). However, several Australian models of out-of-class provision, especially in the primary and junior secondary years, have well-documented success based on careful research and assessment of children's numeracy development. Both forms of provision are discussed in the next section.

Some Australian programs for underperforming students

Research shows that there are students who experience difficulty in learning, and without specific teacher interventions are at risk of longer term underperformance. Gervasoni (in COAG 2008), for example, found that by the end of the first year at school, some 40 per cent of students are at risk in at least one aspect of number learning. Gervasoni argued that at-risk students can lose confidence in their ability, and develop poor attitudes to learning and to school. One outcome is that the gap grows between the knowledge of these children and of other children, and the 'typical' learning experiences provided by the classroom teacher for the class do not enable each child to fully participate and benefit.

The National Numeracy Review (COAG 2008) identified two strands for addressing the needs of at risk students: one is based on structured withdrawal programs; and the other involves addressing students' needs in mainstream classrooms.

Several Australian programs that have been specifically designed to support underperforming students are discussed in this section; however, this report offers no comment on any evaluation of their effectiveness. Some are specifically designed for one-to-one withdrawal, others for small-group withdrawal, and others for in-class support.

Numeracy learning withdrawal programs

One of the best known Australian programs, also adopted internationally, is the Mathematics Recovery Program (Wright, Martland & Stafford 2000) which works intensively with children in the second year of primary school who have been identified as being at risk. It is estimated by the researchers who developed this program that between five and 10 per cent of students may fall into this category.

Children so identified are placed in a long-term intensive individualised teaching program with the aim of having them return to regular classroom teaching after consolidation and rebuilding of their early number understanding. Mathematics Recovery is a one-to-one withdrawal program based on identification of very low-achieving students who appear unable to benefit from regular classroom teaching. Mathematics Recovery provides a highly structured diagnostic assessment of children's mathematical knowledge. It is a resource intensive program with a demonstrated record of success.

Mathematics Intervention (Pearn and Merrifield 1998) is another withdrawal program developed in Victoria. Like Mathematics Recovery, it focuses on students identified as at-risk in the second year of schooling, but it also allows for these students to be withdrawn in small groups. Trained specialist teachers work intensively with students in out-of-class settings.

Extending Mathematical Understanding (Gervasoni 2004) is another Victorian intervention program designed for six- and seven-year-old children who are diagnosed as at-risk in aspects of early number learning. The program uses daily 30-minute lessons conducted over a period of between ten and twenty weeks.

Each of these withdrawal programs employ a similar approach to instruction which involves direct teaching with high-level intensive questioning and scaffolding by specialist teachers.

In junior secondary school, Pegg, Graham and Bellert (2005) used a similar withdrawal approach with low-achieving students aged 11- to 13-years old in a number of NSW schools. Their program, Quicksmart, was based on individual assessment followed by explicit teaching of number facts in regular and extended withdrawal sessions over a period of 25 weeks. Their goal was to achieve speedy responses to number facts and operations (hence, *Quick*) and appropriate use of mathematical strategies (hence, *smart*) from under achieving junior secondary students. The National Numeracy Review report cites evidence of the success of these programs (COAG 2008, pp. 77–78).

Numeracy learning in-class support

An initiative in Western Australia designed to improve outcomes for students in 'high needs schools' is the Getting it Right literacy and numeracy strategy (Cahill 2005). This program trains specialist teachers to work in regular classrooms to identify students who are at risk. In this program, students are not regularly withdrawn.

The key purposes of the strategy are to improve literacy and numeracy outcomes across government schools, and to achieve greater parity of outcomes across all groups of students. The model for teachers' professional learning incorporates many features of effective professional development identified in current research findings. Central to that professional learning has been how to select, collect and analyse credible diagnostic and summative student performance data to inform the teaching and learning cycle, whole-school planning and resource allocation.

Funds are provided to appoint a teacher within a school (determined by its statewide and local assessment data as in need of support) as a specialist teacher. Specialist teachers participate in seven three-day training workshops spaced across the two years of their appointment, the first of which occurs before they begin in the role. They:

work shoulder-to-shoulder in classrooms with colleagues, assisting with the collection and analysis of student performance data, using that data to inform planning, modelling lessons and team-teaching (Cahill 2005).

Principals of participating schools are required to set a two-year school improvement target for literacy or numeracy, then to track and report their school's progress towards that improvement target. The specialist teacher also facilitates implementation of a whole school approach to literacy or numeracy. The basic premise of the Getting it Right literacy and numeracy strategy is that teachers make the difference. Further, that the biggest difference is likely to occur when:

- classroom teachers use assessment to set targets and to monitor achievement
- there is a whole-school approach with clear support from school leadership
- professional development is provided to foster a deep and thorough understanding of the outcomes students are required to learn and to enhance teachers' pedagogical content knowledge
- teachers have high expectations of all students
- teachers, both classroom and specialist, have deep and thorough knowledge of 'where their students are at' in relation to progress towards those outcomes
- teachers, both classroom and specialist, use a broad repertoire of teaching practices from which to select so they can properly match

student learning experiences to student needs (Department of Education and Training Western Australia 2007, cited in COAG 2008, p. 61).

In Victoria, the Teaching and Learning Coaches initiative (DEECD 2008b) seeks to improve the quality of teaching in mathematics to improve student learning outcomes. In 2008–09, two hundred coaches will work with teachers in their classrooms. The initiative will be a focus of the school's Strategic Plan, Annual Implementation Plan and Principal's Performance and Development Plan. The role and impact of the Teaching and Learning Coaches initiative in schools will be monitored through the School Accountability and Improvement Framework and a system-wide evaluation will be undertaken by RMIT University. While the initiative focuses on working with teachers, it is targeted at low-performing schools.

Effective approaches to inclusive teaching

One way to deal with the range of student understandings in the classroom is for teachers to use differentiated instruction. This approach is directed to meeting the needs of all students – in particular it seeks to engage students who may be experiencing difficulty and to improve their understanding. It is also intended to challenge those students who are capable of high-level performance in mathematics.

Waters and Montgomery (2008) describe this approach to teaching a mixed ability class whereby the same content is presented at multiple levels. This is sometimes referred to as responsive teaching. Teachers multi-level or differentiate their instruction through the application of several broad instructional strategies for example, using open-ended tasks, using student-driven tasks, providing supports and challenges within tasks, using flexible grouping, and matching strategies to students.

Differentiated instruction

The challenge for education systems is that effective differentiated instruction requires teachers who can identify where various students are on the continuum of mathematics learning, and can plan and teach a program that will move all students further along this continuum. This is vastly different to teaching the year level standard. Differentiated instruction requires teachers with very high pedagogical content knowledge. It also requires the education system to provide a clear mathematics continuum; mathematics assessment tools that easily plot students onto this continuum, and a comprehensive resource bank of mathematics tasks that teachers can use to reliably advance students (see Waters & Montgomery 2008).

A challenge to implementing differentiated instruction is that schools and teachers attempting this approach may adopt ineffective practices such as:

- streaming across classes or within classes whereby students are locked in to set ability groups
- quantitative responses such as giving 'faster' students more work and 'slower' students less work
- using three levels of textbook: the year level, the previous year level and the following year level
- putting all students onto mastery level individualised programs (often via computer).

To avoid possible misconceptions, it is helpful to first consider what inclusive teaching or differentiated instruction is, and what it is not. The Consortium of Ontario School Boards (2003) offers the following useful clarifications:

Differentiating instruction *does* include:

- using a variety of groupings to meet student needs
- providing alternative instruction/assessment activities
- challenging students at an appropriate level, in light of their readiness, interests, and learning profiles.

Differentiating instruction *does not* include:

- doing something different for every student in the class
- disorderly and undisciplined student activity
- using groups that never change, or isolating struggling students within the class
- never engaging in whole-class activities with all students participating in the same endeavour.

Sullivan, Zevenbergen and Mousley (2005) carried out research in classrooms where teachers reported underachievement and low engagement, to examine whether teachers, if given appropriate resources, could use inclusive teaching to support the learning of at-risk students within a regular class environment. In the model of instruction used in this research, particular attention was given to the choice of tasks, the sequence of tasks used, and especially to how particular tasks could be modified (i.e. adapted) by teachers to engage both low-achieving students and to sustain engagement by higher achieving students. This involved the use of specific prompts (i.e. explicit intervention in re-shaping tasks) for at-risk students and the building of an inclusive classroom community based on peer teaching and support, sharing solution strategies, and high expectations of all students. This research reported on improved levels of engagement and noted that teachers needed considerable support if this model of teaching was to become widespread.

McAdamis (2001) conducted similar research in the USA and reported considerable improvement in the test scores of low-achieving students in one school district as a result of differentiated instruction. Teachers also reported improvement in motivation and engagement among students. This study also pointed to the importance of sustained professional development, mentoring and intensive planning if inclusive teaching is to become part of a school's commitment to supporting the learning of all students without resorting to traditional ability grouping. The study likewise noted that teachers were initially reluctant to change their traditional approaches to dealing with a wide range of achievement and engagement in the regular classroom. Strategies such as peer coaching, action research and lesson study groups and ongoing workshops were important measures for supporting change.

Subban (2006) also reports that key issues for teachers are how to utilise time and resources, together with the need to manage differentiated content and diverse student groups during instruction. Tomlinson (1999) also concluded that further research is needed to investigate teachers' apparent reluctance to adopt new models for more inclusive teaching, in particular, teachers' concerns over classroom management. Sullivan, Zevenbergen and Mousley (2005) also drew attention to the need for teachers to have a deep knowledge

of the curriculum – thorough knowledge of curriculum materials, deep knowledge of the curriculum continuum and a capacity to assess where particular students are at. These concerns are often not stated so clearly by teachers for whom time and management issues appear more pressing.

Victorian initiatives

Teaching Secondary Mathematics (DEECD 2008c) is a resource which exemplifies 'placing the learner at the centre'. Teachers must first identify what students know and understand so that they can plan the most effective teaching and learning strategy. These strategies will build on the students' prior knowledge, skills and behaviours and support all students to develop new knowledge and skills. The Teaching and Learning Coaches Initiative (DEECD 2008b) provides intensive assistance to identified schools to bring about the changes in classroom practice that are necessary to improve student outcomes in mathematics and science.

All teachers play a role in fostering students' mathematics achievement and in making the connections between knowledge and application. Liaison between teachers of mathematics and other school staff can encourage the complementary learning that can occur within other domains. While teachers of mathematics have the primary responsibility to develop student mathematical understandings, there are mathematical demands within other learning domains such as Science and Geography.

Summary

Differentiated teaching is a powerful classroom strategy that aims to increase effective learning for all students. Careful use of tasks and teacher prompts are needed to ensure that all students can engage with and benefit from the mathematics they are learning. Differentiated teaching usually relies on a mix of whole-class and small-group work based on short-term flexible groupings according to particular tasks and learning goals.

For at-risk students, research-based withdrawal programs – working either on an intensive individual basis or in small groups – are able to produce significant improvement in mathematical understanding and attainment. The focus of these programs is building up students' conceptual knowledge so that they can become proficient and engaged in learning and using mathematics. There is also evidence that well-designed and well-supported classroom approaches are also effective. These latter approaches need sustained teacher professional development, but this has the effect of building the capacity of all teachers of mathematics across the school.

7. Purposes of assessment

Black and Wiliam (1998) use the general term assessment to refer to all those activities undertaken by teachers – and by their students in assessing themselves – that provide information to be used as feedback to modify teaching and learning activities.

Classroom assessment has multiple and competing purposes including the motivation of students, the diagnosis of difficulties, certification of achievement, and accountability to the public (Earl 2003).

These many purposes sometimes support one another and at times conflict with one another. What is clear is that it is not possible to use one assessment process to fulfil many purposes. Educators need to make choices in their classrooms every day about how to assess for different purposes (Earl 2003).

The Department of Education and Early Childhood Development considers that assessment for improved student learning and deep understanding requires a range of assessment practices to be used, with three overarching purposes:

- Assessment OF learning – occurs when teachers use evidence of student learning to make judgements on student achievement against goals and standards
- Assessment FOR learning (formative) – occurs when teachers use inferences about student progress to inform their teaching
- Assessment AS learning – occurs when students reflect on and monitor their progress to inform their future learning goals (<http://www.education.vic.gov.au/studentlearning/assessment/default.htm>).

Assessment *of* learning describes the extent to which a student has achieved the learning goals, including the Standards and demonstrates what the student knows and can do. Its purpose is summative and gives an 'overview of previous learning' (Black 1998, p28). This is the assessment that is used to certify learning for reporting to students, the parents and the system. It takes place usually at the end of a unit, a program, a semester or a year of study. It is based on teacher moderation to ensure consistent judgement of student achievement and is supported by examples or evidence of student learning. Assessment of learning can be used to plan for future learning goals.

Assessment *for* learning integrates assessment into the learning and teaching process and establishes the teacher's role in assessment. Through assessment for learning teachers ascertain students' knowledge, perceptions and misconceptions and use this evidence to inform curriculum planning and teaching practice in order to support students to operate at the edge of their competence. Teachers use a range of assessment tools and teaching approaches integrating assessment in the learning and teaching process. Assessment goals are explicit and students are assisted to understand clearly what they are trying to learn and what is expected of them. Assessment is seen positively as supportive of student learning and assisting students to close the gap between their current achievement and the expected goal. Assessment for learning recognises the influence that assessment has on the motivation and self-esteem of students and provides them with constructive feedback. Assessment for learning encourages the active involvement of students in their learning and it depends on teachers' diagnostic skills to make it work (Earl 2003).

Assessment as learning establishes students' roles and responsibilities in relation to their learning and assessment. It engages students in self- and peer-assessment and promotes students' confidence and self-esteem through an understanding of how they learn. Its focus on student reflection on their learning is powerful in building metacognition and an ability to plan for their own future learning goals. In assessment as learning students monitor their learning and use feedback from this monitoring to make adaptations and adjustments to what they understand (Earl 2003). Earl also expresses the view that 'effective assessment empowers students to ask reflective questions and consider a range of strategies for learning and acting. Over time, students move forward in their learning when they can use personal knowledge to construct meaning, have skills of self-monitoring to realize that they don't understand something, and have ways of deciding what to do next' (Earl 2003, p. 25). Assessment as learning emphasises the process of learning as it is experienced by the student.

Assessment of the Victorian Essential Learning Standards requires a mix of summative assessment of learning to determine what the student has achieved and formative assessment to inform the next stage of learning that will occur. Both approaches need to include authentic assessment – students develop and demonstrate the application of their knowledge and skills in real world situations, a practice which promotes and supports the development of deeper levels of understanding. Authentic assessment stems from clear criteria which students help to develop and evaluate (VCAA 2007).

NAPLAN

The first National Assessment Program – Literacy and Numeracy (NAPLAN) tests were conducted in May 2008 for all Years 3, 5, 7 and 9 students in government and non-government schools (MCEETYA 2008). For the first time, all students in the same year level were assessed on the same test items in domains of learning including numeracy.

As students progress through their years of schooling, it will now be possible to monitor how much progress they have made in literacy and numeracy. Individual NAPLAN student reports also show their results against the national average and the middle 60 per cent of students nationally.

Individual student reports contain plain English descriptors of what was assessed in each of the tests. This provides parents, in particular, with valuable information on what a student can typically do. Importantly, NAPLAN can also be used by teachers for diagnostic purposes. At all levels, teachers can refer to this information to gauge the achievement of the most able students, as well as focus on students who have yet to reach the national minimum standard and who may need further support.

Further information on NAPLAN, including the report on the 2008 tests is available on the website at <<http://www.naplan.edu.au/>>.

Assessment in the classroom

The results of standardised testing, such as NAPLAN, can set broad directions for teaching practice, curriculum planning and professional learning needs, whereas classroom assessment gives specific

- connect directly and naturally to the curriculum and to what is being taught
- are very time efficient for teachers to manage and use
- allow every student to show what he/she knows and can do
- are engaging and interesting for students
- allow students to utilise a range of methods and approaches
- provide a measure of choice and openness for students
- allow scope for students to express *their* mathematical thinking
- help students to show connections they can make between concepts they have been taught
- are themselves worthwhile activities for mathematical learning
- show teachers what specific help some students require in a given topic or curriculum area
- are able to show clearly expressed standards of achievement – for example, from ‘little progress’ through to ‘some progress’ and on to ‘fully accomplished’ and allows for ‘goes beyond expectations’
- can be readily compared and discussed among teachers and are able to inform reporting to parents
- are informed by standards of assessment and reporting that permit communication and interpretation across classes and schools (e.g. Victorian Essential Learning Standards).

Such tasks need to be well-designed so that their mathematical content is clear, and teachers are clear about what students are expected to do and what equipment may be needed.

Scoring rubrics and work samples

Classroom assessment tasks are often accompanied by scoring rubrics that enable teachers to make what is being assessed explicit for students and to identify areas where improvements could be made. These rubrics may be in part general in that they describe progress towards learning objectives for a unit of work, where that measurement of progress is needed for reporting purposes. But they may also contain specific information about the range of likely responses to a given task so that teachers can make on-the-spot judgements about where particular students are at; who needs help and what kind of help; what responses should be shared with the whole class so as to share different forms of mathematical thinking and representation, and in this way enhance students’ understanding. A good rubric will enable a student to accurately self-assess.

Carefully presented work samples are another effective way of helping teachers to use scoring rubrics and to anticipate the likely range of students’ responses to a task. However, research has shown that work samples on their own – without clear links to appropriate standards of achievement – are not likely to be an effective or efficient use of teachers’ time. Equally, scoring rubrics with appropriate work samples can lack clarity when teachers are confronted by a wide range of responses to a given task. Good classroom assessment tasks can often be used at a range of year levels where expectations of student performance on the task will change accordingly.

The VCAA Mathematics assessment maps provide teachers with a tool to help them assess student work using the Victorian Essential Learning Standards. They provide a range of annotated student work samples which can be used in conjunction with the progression points

The Standards describe what is essential for Victorian students to achieve from Prep to Year 10 in 16 domains within three overarching strands (Physical, Personal and Social Learning, Discipline-based Learning) and provide a whole-school curriculum planning framework that sets out learning standards for schools to use to plan their teaching and learning programs, including assessment and reporting of student achievement and progress. The Mathematics domain in the Discipline-based strand of the Standards is organised around five dimensions: Number; Space; Measurement, chance and data; Structure; and Working Mathematically.

The Standards are developed across 6 levels from Prep (Level 1) to Year 10 (Level 6) and specify what a student should know and be able to do at the end of a one-year period (Level 1) or two-year (Levels 2–6) period of learning. Complementary sets of related progression points have been developed with respect to the Mathematics standards for each level, and these provide a model for three stages of intermediate progression between levels for each dimension.

The Mathematics Developmental Continuum P–10 was released in 2006 and is aligned to the standards and progression points of the Mathematics domain of the Standards. It is designed to assist all teachers to deepen their understanding of the Mathematics domain; monitor individual student progress; enhance teaching skills to enable purposeful teaching; identify the range of student learning levels within their mathematics classes, and develop a shared language to describe and discuss student progress.

Effective classroom assessment requires a thorough and deep knowledge of the learning continuum. This kind of knowledge is needed to shape the quality of teacher feedback to students: teachers need to know, for example, the purposes of particular assessment tasks, the expected standards of performance, and subsequent learning goals. High-quality feedback involves making explicit to students how they are performing, what they need to attend to and what are the next steps forward.

Victorian examples of assessment for learning

The Mathematics Online Interview supports students' understanding of Levels 1-3 of the Mathematics domain in the Standards. This interview is a tool for use by teachers in a one-to-one situation in Prep to Year 4. To further support teachers, there is an online resource titled *I have done the interview, now what?* which provides links from the assessment items to the most appropriate teaching strategies in the Mathematics Developmental Continuum. A *Fractions and Decimals Online Interview* supports students' understanding in the key areas of fractions, decimals, percentages and ratios, and is aimed primarily at middle year students operating between Levels 3 and 5 of the Standards. The *Learning and Assessment Framework for Multiplicative Thinking* framework, for Levels 4 and 5, is based on the Scaffolding Numeracy in the Middle Years project. It brings together in a hierarchy, the key ideas, strategies and representations of multiplication and division needed to work flexibly and confidently with whole numbers, fractions, decimals, and percentages across a wide range of contexts. *VCAA On Demand Testing* enables teachers to identify quickly what students are able to do and areas where they are experiencing

8. Resources

The Victorian Department of Education and Early Childhood Development produces research-based resources to support teachers to implement the Mathematics domain of the Victorian Essential Learning Standards (the Standards). The Standards, and the resources outlined below, are designed to provide professional learning for teachers to enhance their delivery of effective mathematics teaching and learning. A key feature of these resources is that they place the learner at the centre of curriculum planning and delivery.

Mathematics Developmental Continuum P-10

Published on line in 2006 the Continuum is aligned to the standards and progression points of the Mathematics domain of the Standards. It is designed to assist all teachers to deepen their understanding of the Mathematics domain; monitor individual student progress towards achievement of the Mathematics Standards; enhance teaching skills to enable purposeful teaching; identify the range of student learning levels within their mathematics classes, and develop a shared language to describe and discuss student progress. This resource is being progressively enriched and added to over time as new research into mathematics teaching and learning becomes available.

<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm>

Mathematics Online Interview

The on-line assessment interview supports student understanding of Levels 1–3 of the Mathematics domain. It is a tool for teachers of Prep to Year 4, but is also valuable for any teacher who has students operating at Levels 1–3 of the Standards. The interview is used by teachers in a one-on-one situation to determine a student's existing mathematical knowledge in relation to points of growth. Analysis of the student's responses provides information to use when planning to meet this student's learning needs. The interview was originally developed as part of the Early Numeracy Research Project (1999–2001). To support schools to implement the online interview, a suite of resources has been published online, including a comprehensive teachers' manual, links to the Mathematics Continuum and guided observation notes. To further support teachers in planning learning for students a resource titled 'I have done the Interview, now what ...?' provides links from the assessment items to the most appropriate teaching strategies in the Mathematics Continuum.

<https://www.eduweb.vic.gov.au/MathematicsOnline/>

Fractions and Decimals Online Interview

The Fractions and Decimals Online Interview supports student understanding in the key areas of fractions, decimals, percentages and ratio. The interview is aimed primarily at middle year students operating between the Standards Levels 3 to 5. The interview is conducted with students in a one and one situation. It enables students to demonstrate their mathematical skills and understanding through a wide range of tasks. The interview focuses not only on students' answers but the mathematical thinking used, and provides teachers with a range of profiles indicating student achievement in relation to 'Big Ideas' linked to the Interview tasks. A suite of resources supporting the interview is available via the Mathematics domain page. These include the School User Guide, Frequently Asked Questions,

learning modules each with downloadable explanations and presentations.

<http://www.education.vic.gov.au/studentlearning/teachingresources/math/teachsec/default.htm>

Research projects

The Department continues to support research projects which extend knowledge of effective numeracy teaching practice, and is currently an industry partner in the following Australian research Council Linkage scheme projects.

Examining the relationship between the documented curriculum, classroom tasks, and the learning of mathematics (TTML) (2007–10)

Monash University, Australian Catholic University and the Catholic Education Commission of Victoria have undertaken research into the nature of the learning prompted by different mathematical tasks. The challenge to interpret and implement the Standards is reflected in debate within the mathematics education community on key characteristics of mathematics teaching, especially in the middle years, and the types of tasks that can best assist students in achieving desired mathematical goals. The project will support schools in implementing the Mathematics domain in the Standards.

Supporting personalised learning in secondary schools through the use of Specific Mathematics Assessments that Reveal Thinking (SMART) (2007–09)

The University of Melbourne has designed a project to support personalised learning by delivering online mathematical assessments which reveal students thinking. As well as using intelligent technology to create research based tests, diagnosis of results and teaching suggestions, the project will also investigate the long term use of these resources on student achievement and teacher pedagogical content knowledge and will examine the processes through which secondary teachers use the resources to personalise learning of mathematics.

Science and Mathematics Equipment Grants

Science and Mathematics grants have been provided to all Victorian primary, secondary, P-12 and special schools to support improved teaching and learning in science and mathematics. These grants have been designed to enable schools to purchase additional equipment to improve and modernise mathematics and science teaching, allowing students with engaging experiences with new and emerging technologies.

<http://www.education.vic.gov.au/studentlearning/teachingresources/science/equipsec.htm>

Partnerships with the Commonwealth Department of Education, Employment and Workforce Relations (DEEWR)

Victoria continues to support federal initiatives including Australian School Innovation in Science, Technology and Mathematics (ASISTM), Primary Connections and Science by Doing.

Australian School Innovation in Science, Technology and Mathematics (ASISTM)

In Victoria 91 projects have been funded since 2005 to support school clusters to:

Appendix: Hume Region Numeracy Strategy 2008–09

Hume Region has developed a numeracy strategy which outlines the key approach and strategies to be implemented in Hume Region for school improvement in numeracy in 2008–09. The approach is collegiate based with contact between professionals across the region, networks, clusters and schools taking a high priority.

In defining what numeracy looks like in effective schools, the strategy highlights the following:

- Numeracy describes an individual's performance when using mathematics to solve problems and interpret and produce text, arguments and conclusions. It is context specific.
- Describing levels of numeracy performance is relative to the context. An individual is more or less numerate when solving problems in particular contexts. For example, a person's numeracy performance can vary when cooking, or planning finances, drawing building plans and so on.
- All teachers are responsible for their students' numeracy development. This means teachers should have knowledge of the numeracy demands and opportunities in their subject and respond to an individual's learning needs, particularly for those students who are at risk because of limited numeracy skills. For example, history teachers need to support students to learn to read and understand timelines.
- Numeracy development is enhanced when school mathematics teaching includes thinking, reasoning, strategy development and communication, and is embedded in contexts to allow for deep, connected learning between concepts, knowledge and skills.

Using the following key propositions from the Effective Schools model the school improvement process in numeracy is effective when:

1. The principal and key teaching staff provides strong numeracy leadership

Effective numeracy leaders:

- understand the elements of an effective numeracy classroom
- recognise and articulate the essential elements of effective numeracy teaching
- promote and engage in classroom actioned professional learning with teachers
- provide teachers with genuine opportunities to participate directly in decisions about numeracy curriculum, professional learning priorities, and resource management.

2. There is a shared vision and goals for numeracy teaching and learning

- Effective schools have an agreed understanding of effective numeracy practices.
- Teachers and leadership are able to articulate their numeracy practices.
- Goals are set for students' numeracy learning.

3. There is a focus on the teaching and learning of numeracy

- Uninterrupted daily blocks of time are dedicated to numeracy.
- Planning pays careful attention to deep, connected numeracy concepts, knowledge and skills.
- Thinking, reasoning, strategy development, visualisation and communication are integral to deep, connected numeracy development.
- Professional learning focuses on developing the capacities of all teachers through classroom actioned models such as lesson study, case study and action learning.
- Numeracy demands across the curriculum are acknowledged and supported by all teachers at all stages of schooling: Early, Middle and Later Years.

4. Numeracy teaching is purposeful

Effective numeracy teachers:

- know how students learn numeracy, and build on the numeracy knowledge students already have
- have a clear understanding of typical developmental pathways for numeracy.
- teach differentiated lessons that focus on deep, connected numeracy understandings
- help their students make connections between school and 'out of school' numeracy practices.

5. There are high expectations for numeracy learning

Effective teachers:

- use focused assessment and teaching to match learning activities to each student
- set challenging, yet realistic goals with the effort required to succeed made clear
- encourage parents to be actively engaged in their children's learning.

6. Numeracy learning communities are developed

Teachers in effective schools:

- work collaboratively to enhance the numeracy curriculum, teaching strategies and assessment
- foster openness, dialogue, inquiry, risk taking and trust
- keep students' numeracy performance at the centre of school improvement
- inform their decisions through collection and analysis of school numeracy data
- undertake professional learning opportunities at school, cluster, network and regional level which reflect the Principles of Highly Effective Professional Learning (Department of Education and Training 2005) and use models such as peer observation, lesson study, action learning, case study, examining students' work, and study groups.

7. Numeracy monitoring and accountability practices are timely and integrated

Effective schools:

- establish data tracking systems by which school and student performance can be evaluated

- use performance data to identify, support and monitor under-achieving and high-potential students
- use performance data to plan professional learning activities for staff
- monitor and assess student progress regularly to inform planning and to cater for individual student needs
- have a thorough knowledge of student learning and numeracy developmental pathways
- link all these factors together to plan for focused instruction
- have strong accountability measures which allow them to provide parents with meaningful information about their children's progress.

8. Schools provide a stimulating and secure numeracy learning environment

- A positive learning environment supports and challenges students to be risk takers.
- Teachers plan rich and authentic tasks related to students' development.
- Student and classroom management strategies create an environment that is conducive to student learning.
- Strategies for independent learning such as goal setting, reflection and summary of thinking are built into every lesson.
- There is the inclusion of student voice in decision-making.

The strategy details an action plan to achieve improved numeracy outcomes based on these propositions, together with suggestions for the essential elements school clusters need to support numeracy learning including:

- consistent curriculum design that focuses on deep, connected numeracy learning
- personalisation practices to track and provide for every student in numeracy
- alignment of numeracy practices between schools in the cluster
- use of consistent developmental numeracy pathways
- assessment practices which identify the numeracy understandings and needs of all students
- effective professional learning team practices
- accountability processes linked to numeracy improvement
- actively engage the wider school community.

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